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ALGEBRA

PROBLEMS

PROVEN TO IMPROVE ALGEBRA SKILLS

Step by Step
SOLUTIONS BOOK

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MasterAlgebraNow

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SECTION 1 : SUBSTITUTING VALUES

Practice 1

Determine the value of $7 - 5y$ when $y = 8$.

$7 - 5y$

$= 7 - 5(8)$

$= 7 - 40$

$= -33$

Practice 2

Determine the value of $\frac{4p^3}{5}$ when $p = 5$.

$$\frac{4p^3}{5}$$

$$= \frac{4(5)^3}{5}$$

$$= \frac{4(125)}{5}$$

$$= 100$$

Practice 3

Determine the value of $ab - 8$ if $a = 5$ and $b = -2$.

$ab - 8$

$= (5)(-2) - 8$

$= -10 - 8$

$= -18$

Practice 4

Find the value of $p^2 + 4p - 8$ when $p = -3$.

$p^2 + 4p - 8$

$= (-3)^2 + 4(-3) - 8$

$= 9 - 12 - 8$

$= -11$

Practice 5

Find the value of $4(5 - 7p)$ when $p = \frac{2}{21}$

$4(5 - 7p)$

$= 4\left(5 - 7 \times \frac{2}{21}\right)$

$= 4\left(5 - \frac{2}{3}\right)$

$= 4\left(4\frac{1}{3}\right)$

$= 17\frac{1}{3}$

Practice 6

Determine the value of the algebraic expression $(2 - 3p)^2$ if $p = -1$.

$(2 - 3p)^2$

$= (2 - 3(-1))^2$

$= (2 + 3)^2$

$= 5^2$

$= 25$

Practice 7

Determine the value of the algebraic expression $(4a + b)^2$ if $a = -1$ and $b = 4$.

$(4a + b)^2$

$= (4(-1) + 4)^2$

$= 0^2$

$= 0$

Practice 8

Find the value of $(-1 + 8x)(5 + 7x)$ when $x = 3$

$(-1 + 8x)(5 + 7x)$

$= \left(-1 + 8(3)\right)\left(5 + 7(3)\right)$

$= (-1 + 24)(5 + 21)$

$= (23)(26)$

$= 598$

Practice 9

Find the value of $\sqrt{\frac{3x-4}{2}} + 2x$ when $x = 18$

$\sqrt{\frac{3x-4}{2}} + 2x$

$= \sqrt{\frac{3(18)-4}{2}} + 2(18)$

$= \sqrt{25} + 36$

$= 5 + 36$

$= 41$

Practice 10

Determine the value of $\sqrt[3]{4m^2} - m$ when $m = 2$

$\sqrt[3]{4m^2} - m$

$= \sqrt[3]{4(2)^2} - 2$

$= \sqrt[3]{16} - 2$

$= 0.51984$

≈ 0.520 (3s.f.)

SECTION 2 : EXPANSION

Practice 1

Expand $3(2a - 7)$

$3(2a - 7)$

$= 6a - 21$

Practice 2

Expand $-4(8 - 5x)$

$-4(8 - 5x)$

$= -32 + 20x$

Practice 3

Expand $(2p + 5)^2$

$(2p + 5)^2$

$= (2p)^2 + 2(2p)(5) + 5^2$

$= 4p^2 + 20p + 25$

Practice 4

Expand $(4x - 3)^2$

$(4x - 3)^2$

$= (4x)^2 - 2(4x)(3) + 3^2$

$= 16x^2 - 24x + 9$

Practice 5

Expand and simplify $(4x - 3)(6 - 11x)$

$(4x - 3)(6 - 11x)$

$= (4x)(6) + (-3)(6) + (4x)(-11x) + (-3)(-11x)$

$= 24x - 18 - 44x^2 + 33x$

$= -44x^2 + 57x - 18$

Practice 6

Expand and simplify $-2(4x - 3)(6 - 11x)$

$-2(4x - 3)(6 - 11x)$

$= -2(4x - 3)(6 - 11x)$

$= -2[(4x)(6) + (-3)(6) + (4x)(-11x) + (-3)(-11x)]$

$= -2[24x - 18 - 44x^2 + 33x]$

$= -2[-44x^2 + 57x - 18]$

$= 88x^2 - 114x + 36$

Practice 7

Expand and simplify $(x + 2)(5 - 8x) + (5 - 4x)(2x - 1)$

$(x + 2)(5 - 8x) + (5 - 4x)(2x - 1)$

$= (x)(5) + (2)(5) + (x)(-8x) + (2)(-8x) + (5)(2x) + (-4x)(2x) + (5)(-1) + (-4x)(-1)$

$= 5x + 10 - 8x^2 - 16x + 10x - 8x^2 - 5 + 4x$

$= -16x^2 + 3x + 5$

Practice 8

Expand and simplify $(x - 2)(1 - 7x) - (2 - 3x)(2x + 11)$

$(x - 2)(1 - 7x) - (2 - 3x)(2x + 11)$

$= (x)(1) + (-2)(1) + (x)(-7x) + (-2)(-7x) - [(2)(2x) + (-3x)(2x) + (2)(11) + (-3x)(11)]$

$= x - 2 - 7x^2 + 14x - [4x - 6x^2 + 22 - 33x]$

$= -7x^2 + 15x - 2 - [6x^2 - 29x + 22]$

$= -7x^2 + 15x - 2 + 6x^2 + 29x - 22$

$= -x^2 + 44x - 24$

Practice 9

Expand and simplify $(4p + 3)(3 - p) - 2(1 - 2p)^2$

$(4p + 3)(3 - p) - 2(1 - 2p)^2$

$= (4p + 3)(3 - p) - 2(1 - 2p)^2$

$= (4p)(3) + (3)(3) + (4p)(-p) + 3(-p) - 2[(1)^2 - 2(1)(2p) + (2p)^2]$

$= 12p + 9 - 4p^2 - 3p - 2[1 - 4p + 4p^2]$

$= -4p^2 + 9p + 9 - 2 + 8p - 8p^2$

$= -12p^2 + 17p + 7$

Practice 10

Given that the value of $x^2 + \frac{1}{x^2} = 10$, find the value(s) of $x - \frac{1}{x}$

$(x - \frac{1}{x})^2 = x^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$

$(x - \frac{1}{x})^2 = x^2 - 2 + \frac{1}{x^2}$

$x - \frac{1}{x} = \pm\sqrt{x^2 - 2 + \frac{1}{x^2}}$

$$= \pm\sqrt{10 - 2}$$

$$= \pm\sqrt{8}$$

$$= \pm 2.8284$$

$$\approx \pm 2.83 \text{ (3 s.f.)}$$

SECTION 3 : FACTORISATION

Practice 1

Factorise $8w - 24$

$8w - 24$

$= 8(w - 3)$

Practice 2

Factorise $8aw^2 - 24a^3w$

$8aw^2 - 24a^3w$

$= 8aw(w - 3a^2)$

Practice 3

Factorise $6p^2 + 13p - 8$

$6p^2 + 13p - 8$

$$(3p + 8)(2p - 1)$$

Practice 4

Factorise $24x^2 + 16x - 14$

$24x^2 + 16x - 14$

$= 2(12x^2 + 8x - 7)$

$= 2(2x - 1)(6x + 7)$

Practice 5

Factorise $6x^2 + 5xy - 6y^2$

$$
\begin{array}{c|c}
3x \quad -2y & -4xy \\
2x \quad +3y & +9xy \\
\hline
6x^2 \quad -6y^2 & +5xy
\end{array}
$$

$$(3x - 2y)(2x + 3y)$$

Practice 6

Factorise $2y^2 + 3y - 5$ and hence factorise $2(x + 1)^2 + 3(x + 1) - 5$

$$
\begin{array}{c|c}
2y \quad +5 & +5y \\
y \quad -1 & -2y \\
\hline
2y^2 \quad -5 & +3y
\end{array}
$$

$$2y^2 + 3y - 5$$

$= (2y + 5)(y - 1) \ldots \ldots Eqn.1$

$$2(x + 1)^2 + 3(x + 1) - 5$$

Changing y to $x + 1$ in Eqn.1

$2(x + 1)^2 + 3(x + 1) - 5 = (2(x + 1) + 5)((x + 1) - 1)$

$$= (2x + 2 + 5)(x)$$

$$= x(2x + 7)$$

Practice 7

Factorise $16 - p^2$

$16 - p^2$

$= 4^2 - p^2$

$= (4 + p)(4 - p)$

Practice 8

Factorise $4 - 16a^2$

$4 - 16a^2$

$= 4(1 - 4a^2)$

$= 4(1^2 - (2a)^2)$

$= 4(1 + 2a)(1 - 2a)$

Practice 9

Factorise $4x^3 + x^2 - 4x - 1$

$4x^3 + x^2 - 4x - 1$

$= x^2(4x + 1) - 1(4x + 1)$

$= (x^2 - 1)(4x + 1)$

$= (x^2 - 1^2)(4x + 1)$

$= (x + 1)(x - 1)(4x + 1)$

Practice 10

Factorise $6ap - 5q + 15p - 2aq$

$6ap - 5q + 15p - 2aq$

$= 6ap - 2aq - 5q + 15p$

$= 2a(3p - q) - 5(q - 3p)$

$= 2a(3p - q) + 5(3p - q)$

$= (2a + 5)(3p - q)$

SECTION 4 : SIMPLIFICATION

Practice 1

Simplify $4x - 9 + x - 10$

$4x - 9 + x - 10$

$= 5x - 19$

Practice 2

Simplify $3p - 9p^2 + p - 10p^2 - 1 + 8$

$3p - 9p^2 + p - 10p^2 - 1 + 8$

$= -19p^2 + 4p + 7$

Practice 3

Simplify $\dfrac{6}{3p-1} + \dfrac{2}{1-3p}$

$$\dfrac{6}{3p-1} + \dfrac{2}{1-3p}$$

$$= \dfrac{6}{3p-1} - \dfrac{2}{3p-1}$$

$$= \dfrac{6-2}{3p-1}$$

$$= \dfrac{4}{3p-1}$$

Practice 4

Simplify $\dfrac{6}{3p-1} - \dfrac{2}{1-3p}$

$$\dfrac{6}{3p-1} - \dfrac{2}{1-3p}$$

$$= \dfrac{6}{3p-1} + \dfrac{2}{3p-1}$$

$$= \dfrac{6+2}{3p-1}$$

$$= \dfrac{8}{3p-1}$$

Practice 5

Simplify $\dfrac{6}{3p-1} - \dfrac{2}{(1-3p)^2}$

$$\dfrac{6}{3p-1} - \dfrac{2}{(1-3p)^2}$$

$$= -\dfrac{6}{1-3p} - \dfrac{2}{(1-3p)^2}$$

$$= -\dfrac{6(1-3p)}{(1-3p)^2} - \dfrac{2}{(1-3p)^2}$$

$$= \dfrac{-6+18p}{(1-3p)^2} - \dfrac{2}{(1-3p)^2}$$

$$= \dfrac{-6+18p-2}{(1-3p)^2}$$

$$= \frac{18p-8}{(1-3p)^2}$$

$$= \frac{2(9p-4)}{(1-3p)^2}$$

Practice 6

Simplify $\frac{4}{2x-1} - \frac{3}{(1+2x)(1-2x)}$

$$\frac{4}{2x-1} - \frac{3}{(1+2x)(1-2x)}$$

$$= \frac{4}{2x-1} + \frac{3}{(1+2x)(2x-1)}$$

$$= \frac{4(1+2x)}{(2x-1)(1+2x)} + \frac{3}{(1+2x)(2x-1)}$$

$$= \frac{4+8x+3}{(2x-1)(1+2x)}$$

$$= \frac{7+8x}{(2x-1)(1+2x)}$$

Practice 7

Simplify $\frac{3w}{4} - \frac{2w}{5}$

$$\frac{3w}{4} - \frac{2w}{5}$$

$$= \frac{3w(5)}{4(5)} - \frac{2w(4)}{5(4)}$$

$$= \frac{15w-8w}{20}$$

$$= \frac{7w}{20}$$

Practice 8

Simplify $\frac{3}{16x^2-9} - \frac{1}{8x^2-2x-3}$

$$\frac{3}{16x^2-9} - \frac{1}{8x^2-2x-3}$$

$$= \frac{3}{(4x)^2-3^2} - \frac{1}{8x^2-2x-3}$$

$$= \frac{3}{(4x+3)(4x-3)} - \frac{1}{(4x-3)(2x+1)}$$

$$
\begin{array}{cc|c}
4x & -3 & -6x \\
2x & +1 & +4x \\
\hline
8x^2 & -3 & -2x
\end{array}
$$

$$= \frac{3(2x+1)}{(4x+3)(4x-3)(2x+1)} - \frac{1(4x+3)}{(4x-3)(2x+1)(4x+3)}$$

$$= \frac{6x+3-4x-3}{(4x+3)(4x-3)(2x+1)}$$

$$= \frac{2x}{(4x+3)(4x-3)(2x+1)}$$

Practice 9

Simplify $\frac{4x+2}{2x^2-9x-5} \div \frac{8}{2x^2-50}$

$$\frac{4x+2}{2x^2-9x-5} \div \frac{8}{2x^2-50}$$

$$= \frac{2(2x+1)}{2x^2-9x-5} \div \frac{8}{2(x^2-25)}$$

$$= \frac{2(2x+1)}{(2x+1)(x-5)} \div \frac{8}{2(x^2-5^2)}$$

$$= \frac{2(2x+1)}{(2x+1)(x-5)} \div \frac{8}{2(x+5)(x-5)}$$

$$= \frac{2(2x+1)}{(2x+1)(x-5)} \times \frac{2(x+5)(x-5)}{8}$$

$$= \frac{x+5}{2}$$

Practice 10

Simplify $\frac{7p}{4-16p} - \frac{2}{16p^2-1}$

$$\frac{7p}{4-16p} - \frac{2}{16p^2-1}$$

$$= \frac{7p}{4(1-4p)} - \frac{2}{(4p)^2-1^2}$$

$$= \frac{7p}{4(1-4p)} - \frac{2}{(4p+1)(4p-1)}$$

$$= \frac{7p}{4(1-4p)} + \frac{2}{(4p+1)(1-4p)}$$

$$= \frac{7p(4p+1)}{4(1-4p)(4p+1)} + \frac{2(4)}{(4p+1)(1-4p)(4)}$$

$$= \frac{28p^2+7p+8}{4(1-4p)(4p+1)}$$

REVIEW PRACTICE 1

(Substitution, Expansion, Factorisation and Simplification)

Q1. Find the value of the following algebraic expression if the value of $g = -7$

$$\frac{3g^2}{7} + 8$$

$\frac{3g^2}{7} + 8$

$= \frac{3g^2}{7} + 8$

$= \frac{3(-7)^2}{7} + 8$

$= \frac{3(49)}{7} + 8$

$= 21 + 8$

$= 29$

Q2. Expand and simplify $-3(4x - 1) + 5(2 + 7x)$

$-3(4x - 1) + 5(2 + 7x)$

$= -12x + 3 + 10 + 35x$

$= 23x + 13$

Q3. Factorise the following expression completely

$18p^2 - 24p - 64$

$= 2(9p^2 - 12p - 32)$

$= 2(3p + 4)(3p - 8)$

$$
\begin{array}{ll|l}
3p & +4 & +12p \\
3p & -8 & -24p \\
\hline
9p^2 & -32 & -12p
\end{array}
$$

Q4. Simplify the following algebraic expression

$\dfrac{4a}{5} - \dfrac{8}{10a}$

$= \dfrac{4a(2a)}{5(2a)} - \dfrac{8}{10a}$

$= \dfrac{8a^2}{10a} - \dfrac{8}{10a}$

$= \dfrac{8a^2 - 8}{10a}$

$$= \frac{8(a^2 - 1)}{10a}$$

$$= \frac{8(a^2 - 1^2)}{10a}$$

$$= \frac{4(a + 1)(a - 1)}{5a}$$

Q5. Factorise the following algebraic expression completely $(x - 1)^3 - 100x + 100$

$(x - 1)^3 - 100x + 100$

$= (x - 1)^3 - 100(x - 1)$

$= (x - 1)[(x - 1)^2 - 100]$

$= (x - 1)[(x - 1)^2 - 10^2]$

$= (x - 1)[(x - 1) + 10] [(x - 1) - 10]$

$= (x - 1)(x + 9)(x - 11)$

Q6. Expand and simplify $-2(3x + 1)^2$

$-2(3x + 1)^2$

$= -2[(3x)^2 + 2(3x)(1) + (1)^2]$

$= -2[9x^2 + 6x + 1]$

$= -18x^2 - 12x - 2$

Q7. Factorise the following algebraic expression completely

$2a - 8a^3$

$2a - 8a^3$

$= 2a(1 - 4a^2)$

$= 2a((1)^2 - (2a)^2)$

$= 2a(1 + 2a)(1 - 2a)$

Q8. Factorise $6x^3 - 16x^2 + 3x - 8$

$6x^3 - 16x^2 + 3x - 8$

$= 2x^2(3x - 8) + 1(3x - 8)$

$= (2x^2 + 1)(3x - 8)$

Q9. Given that $x^2 + 10x = -25$, find the value of $x + 5$ without solving for x.

$x^2 + 10x = -25$

$x^2 + 10x + 25 = 0$

$(x + 5)^2 = 0$

$x + 5 = 0$

Q10. Simplify the following algebraic expression completely.

$$\frac{2}{4x^2 - 1} - \frac{5}{1 - 2x}$$

$$= \frac{2}{(2x)^2 - 1^2} - \frac{5}{1 - 2x}$$

$$= \frac{2}{(2x + 1)(2x - 1)} + \frac{5}{2x - 1}$$

$$= \frac{2}{(2x + 1)(2x - 1)} + \frac{5(2x + 1)}{(2x - 1)(2x + 1)}$$

$$= \frac{2 + 10x + 5}{(2x + 1)(2x - 1)}$$

$$= \frac{10x + 7}{(2x + 1)(2x - 1)}$$

SECTION 5 : COMPLETING THE SQUARE

Practice 1

Complete the square for $x^2 + 4x - 1$

$x^2 + 4x - 1$

$= (x + \frac{4}{2})^2 - (\frac{4}{2})^2 - 1$

$= (x + 2)^2 - (2)^2 - 1$

$= (x + 2)^2 - 4 - 1$

$= (x + 2)^2 - 5$

Practice 2

Complete the square for $x^2 + x$

$x^2 + x$

$= (x + \frac{1}{2})^2 - (\frac{1}{2})^2$

$= (x + \frac{1}{2})^2 - \frac{1}{4}$

Practice 3

Express $x^2 - 12x + 8$ in the form $a(x - h)^2 + k$. Hence, state the values of a, h and k.

$x^2 - 12x + 8$

$= (x - \frac{12}{2})^2 - (-\frac{12}{2})^2 + 8$

$= (x - 6)^2 - 36 + 8$

$= (x - 6)^2 - 28$

$a = 1, h = 6, k = -28$

Practice 4

Complete the square for the quadratic equation $-x^2 + x - 1$. Hence, state the maxima coordinates of this equation.

$-x^2 + x - 1$

$= -(x^2 - x + 1)$

$= -[(x - \frac{1}{2})^2 - (-\frac{1}{2})^2 + 1]$

$= -[(x - \frac{1}{2})^2 - \frac{1}{4} + 1]$

$$= -[(x - \frac{1}{2})^2 + \frac{3}{4}]$$

$$= -(x - \frac{1}{2})^2 - \frac{3}{4}$$

Maxima Coordinates $= (\frac{1}{2}, -\frac{3}{4})$

Practice 5

If $3x^2 - 12x + 6 = p(x + h)^2 + k$, state the values of p, h and k.

$3x^2 - 12x + 6$

$= 3[x^2 - 4x + 2]$

$= 3[(x - \frac{4}{2})^2 - (-\frac{4}{2})^2 + 2]$

$= 3[(x - 2)^2 - 4 + 2]$

$= 3[(x - 2)^2 - 2]$

$= 3(x - 2)^2 - 6$

$p = 3, h = -2, k = -6$

Practice 6

Complete the square for the quadratic equation $x^2 - x$. Hence, state the minima coordinates of this equation.

$x^2 - x$

$= (x - \frac{1}{2})^2 - (-\frac{1}{2})^2$

$= (x - \frac{1}{2})^2 - \frac{1}{4}$

Minima Coordinates $= (\frac{1}{2}, -\frac{1}{4})$

Practice 7

Express $x^2 - \frac{1}{2}x + \frac{1}{2}$ in the form $(x + h)^2 - k$.

$x^2 - \frac{1}{2}x + \frac{1}{2}$

$= (x - \frac{1}{4})^2 - (-\frac{1}{4})^2 + \frac{1}{2}$

$= (x - \frac{1}{4})^2 - \frac{1}{16} + \frac{1}{2}$

$$= (x - \frac{1}{4})^2 + \frac{7}{16}$$

Practice 8

Find the minima value of $x^2 - 2x + 1$

$x^2 - 2x + 1$

$$= (x - \frac{2}{2})^2 - (-\frac{2}{2})^2 + 1$$

$$= (x - 1)^2 - 1 + 1$$

$$= (x - 1)^2$$

Minima Value = 0

Practice 9

Find the maxima value of $-2x^2 + 8x$

$-2x^2 + 8x$

$$= -2(x^2 - 4x)$$

$$= -2[(x - \frac{4}{2})^2 - (-\frac{4}{2})^2]$$

$$= -2[(x - 2)^2 - 4]$$

$$= -2(x - 2)^2 + 8$$

Maxima Value = 8

Practice 10

Find the maxima value of $-10x^2 - x + 5$

$-10x^2 - x + 5$

$$= -10[x^2 + \frac{1}{10}x - \frac{1}{2}]$$

$$= -10[(x + \frac{1}{20})^2 - \left(\frac{1}{20}\right)^2 - \frac{1}{2}]$$

$$= -10[(x + \frac{1}{20})^2 - \frac{1}{400} - \frac{1}{2}]$$

$$= -10[(x + \frac{1}{20})^2 - \frac{201}{400}]$$

$$= -10(x + \frac{1}{20})^2 + 5\frac{1}{40}$$

Maxima Value $= 5\frac{1}{40}$

SECTION 6 : SOLVING (LINEAR)

Practice 1

Solve $5p + 25 = 0$

$5p + 25 = 0$

$5p = -25$

$p = -\dfrac{25}{5}$

$p = -5$

Practice 2

Solve $2a + 2 = 8a - 4$

$2a + 2 = 8a - 4$

$2 + 4 = 8a - 2a$

$6 = 6a$

$6a = 6$

$a = \dfrac{6}{6}$

$a = 1$

Practice 3

Solve the following : $-(x + 6) = 3(4 + 5x)$

$-(x + 6) = 3(4 + 5x)$

$-x - 6 = 12 + 15x$

$-18 = 15x + x$

$-18 = 16x$

$16x = -18$

$x = -\dfrac{18}{16}$

$x = -\dfrac{9}{8}$

$x = -1\dfrac{1}{8}$

Practice 4

Solve the following : $4(a - 6) - 2(4 + a) = 6a + 7$

$4(a - 6) - 2(4 + a) = 6a + 7$

$4a - 24 - 8 - 2a = 6a + 7$

$2a - 32 = 6a + 7$

$-32 - 7 = 6a - 2a$

$-39 = 4a$

$4a = -39$

$a = -\dfrac{39}{4}$

$a = -9\dfrac{3}{4}$

Practice 5

Find the value of the unknown in $\frac{2x-1}{3} = 5$

$\dfrac{2x - 1}{3} = 5$

$2x - 1 = 5 \times 3$

$2x - 1 = 15$

$2x = 15 + 1$

$2x = 16$

$x = \dfrac{16}{2}$

$x = 8$

Practice 6

Solve $\frac{3p}{2} + 25 = 0$

$\dfrac{3p}{2} + 25 = 0$

$\dfrac{3p}{2} = -25$

$3p = -25 \times 2$

$$3p = -50$$

$$p = -\frac{50}{3}$$

$$p = -16\frac{2}{3}$$

Practice 7

Solve $\frac{2(x-8)}{3} - 7 = 0$

$$\frac{2(x-8)}{3} - 7 = 0$$

$$\frac{2(x-8)}{3} = 7$$

$$2(x-8) = 7 \times 3$$

$$2(x-8) = 21$$

$$2x - 16 = 21$$

$$2x = 21 + 16$$

$$2x = 37$$

$$x = \frac{37}{2}$$

$$x = 18\frac{1}{2}$$

Practice 8

Find the value of the unknown in $\frac{5+x}{2} + \frac{4-x}{3} = 0$

$$\frac{5+x}{2} + \frac{4-x}{3} = 0$$

$$\frac{(5+x)(3)}{2(3)} + \frac{(4-x)(2)}{3(2)} = 0$$

$$\frac{15+3x}{6} + \frac{8-2x}{6} = 0$$

$$\frac{15+3x+8-2x}{6} = 0$$

$$\frac{23+x}{6} = 0$$

$23 + x = 0$

$x = -23$

Practice 9

Solve for b in the following equation $\frac{3b-1}{4} - \frac{4-b}{3} = 2$

$$\frac{(3b-1)(3)}{4(3)} - \frac{(4-b)(4)}{3(4)} = 2$$

$$\frac{9b-3}{12} - \frac{16-4b}{12} = 2$$

$$\frac{9b-3-16+4b}{12} = 2$$

$$\frac{13b-19}{12} = 2$$

$$13b - 19 = 24$$

$$13b = 24 + 19$$

$$13b = 43$$

$$b = \frac{43}{13}$$

$$b = 3\frac{4}{13}$$

Practice 10

Solve $\frac{3(x+1)}{2} - \frac{4(3-2x)}{5} = -3$

$$\frac{3(x+1)}{2} - \frac{4(3-2x)}{5} = -3$$

$$\frac{3(x+1)(5)}{10} - \frac{4(3-2x)(2)}{10} = -3$$

$$\frac{15x+15}{10} - \frac{24-16x}{10} = -3$$

$$\frac{15x+15-24+16x}{10} = -3$$

$$\frac{31x-9}{10} = -3$$

$$31x - 9 = -3 \times 10$$

$$31x - 9 = -30$$

$$31x = -30 + 9$$

$$31x = -21$$

$$x = -\frac{21}{31}$$

SECTION 7 : SOLVING (QUADRATIC)

Practice 1

Solve the following $x^2 - 3x - 10 = 0$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$(x - 5) = 0 \quad or \quad (x + 2) = 0$

$x = 5 \quad or \quad x = -2$

Practice 2

Solve the following quadratic equation

$x^2 - 10x + 25 = 0$

$x^2 - 10x + 25 = 0$

$(x - 5)^2 = 0$

$x - 5 = \pm\sqrt{0}$

$x = 5$

Practice 3

Solve by using the quadratic formula $2x^2 - 8x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{8 \pm \sqrt{64 - 24}}{4}$$

$$= \frac{8 \pm \sqrt{40}}{4}$$

$= \dfrac{8+\sqrt{40}}{4}$ or $\dfrac{8-\sqrt{40}}{4}$

$= 3.5811$ or 0.4188

≈ 3.58 or 0.419 (3 s.f.)

Practice 4

By completing the square, solve $x^2 - 3x + 2 = 0$

$x^2 - 3x + 2 = 0$

$(x - \dfrac{3}{2})^2 - \left(-\dfrac{3}{2}\right)^2 + 2 = 0$

$(x - \dfrac{3}{2})^2 - 2.25 + 2 = 0$

$(x - \dfrac{3}{2})^2 - 0.25 = 0$

$(x - \dfrac{3}{2})^2 = 0.25$

$x - \dfrac{3}{2} = \pm\sqrt{0.25}$

$x = \dfrac{3}{2} \pm \sqrt{0.25}$

$x = \dfrac{3}{2} + \dfrac{1}{2}$ or $x = \dfrac{3}{2} - \dfrac{1}{2}$

$x = 2$ or $x = 1$

Practice 5

Solve $(3x - 2)^2 = 9$

$(3x - 2)^2 = 9$

$3x - 2 = \pm\sqrt{9}$

$3x = 2 \pm \sqrt{9}$

$3x = 2 + \sqrt{9}$ or $3x = 2 - \sqrt{9}$

$x = \dfrac{2+\sqrt{9}}{3}$ or $x = \dfrac{2-\sqrt{9}}{3}$

$x = \dfrac{2+\sqrt{9}}{3}$ or $x = \dfrac{2-\sqrt{9}}{3}$

$$x = \frac{5}{3} \qquad \text{or} \qquad x = -\frac{1}{3}$$

$$x = 1\frac{2}{3}$$

Practice 6

Solve the following algebraic expression

$$\frac{5}{x} - \frac{x}{5} = 1$$

$$\frac{5}{x} - \frac{x}{5} = 1$$

$$\frac{5(5)}{5x} - \frac{x(x)}{5x} = 1$$

$$\frac{25}{5x} - \frac{x^2}{5x} = 1$$

$$\frac{25 - x^2}{5x} = 1$$

$$25 - x^2 = 5x$$

$$-x^2 - 5x + 25 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-1)(25)}}{2(-1)}$$

$$= \frac{5 \pm \sqrt{25 + 100}}{-2}$$

$$= \frac{5 \pm \sqrt{125}}{-2}$$

$$x = \frac{5 + \sqrt{125}}{-2} \qquad \text{or} \qquad \frac{5 - \sqrt{125}}{-2}$$

$$x = -8.0901 \qquad \text{or} \qquad 3.0901$$

$$x \approx -8.09 \qquad \text{or} \qquad 3.09 \qquad \text{(3 s.f.)}$$

Practice 7

Solve $6x^2 - 7x - 20 = 0$

$(3x + 4)(2x − 5) = 0$

$(3x + 4) = 0$ or $(2x − 5) = 0$

$3x = −4$ $2x = 5$

$x = −\dfrac{4}{3}$ $x = \dfrac{5}{2}$

$x = −1\dfrac{1}{3}$ $x = 2\dfrac{1}{2}$

Practice 8

The length of a rectangular garden is 6 *m* more than it's width .

The area of the garden is 27 m². Find the dimensions of the garden.

Let the width be x, then the length will be $x + 6$
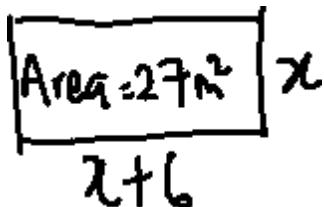
Area $= 27m^2$

$x(x + 6) = 27$

$x^2 + 6x − 27 = 0$

$(x + 9)(x − 3) = 0$

$(x + 9) = 0$ or $(x − 3) = 0$

$x = −9 (rejected\ since\ x > 0)$ or $x = 3$

Therefore, width = 3m, and length = 3 + 6 = 9m

Practice 9

Express *x* in terms of *y* for $81x^2 − 144xy + 64y^2 = 0$

$$81x^2 − 144xy + 64y^2 = 0$$

$$(9x - 8y)^2 = 0$$

$$9x - 8y = 0$$

$$9x = 8y$$

$$x = \frac{8y}{9}$$

Practice 10

Solve this equation by completing the square

$-3x^2 + 5x + 10 = 0$

$-3x^2 + 5x + 10$

$= -3[x^2 - \frac{5}{3}x - \frac{10}{3}]$

$= -3[(x - \frac{5}{6})^2 - \left(-\frac{5}{6}\right)^2 - \frac{10}{3}]$

$= -3[(x - \frac{5}{6})^2 - \frac{25}{36} - \frac{10}{3}]$

$= -3[(x - \frac{5}{6})^2 - \frac{145}{36}]$

$-3(x - \frac{5}{6})^2 + \frac{145}{12} = 0$

$-3(x - \frac{5}{6})^2 = -\frac{145}{12}$

$3(x - \frac{5}{6})^2 = \frac{145}{12}$

$(x - \frac{5}{6})^2 = \frac{145}{36}$

$x - \frac{5}{6} = \pm\sqrt{\frac{145}{36}}$

$x = \frac{5}{6} \pm \sqrt{\frac{145}{36}}$

$x = \frac{5}{6} + \sqrt{\frac{145}{36}}$ or $\frac{5}{6} - \sqrt{\frac{145}{36}}$

$= 2.8402$ or -1.1736

≈ 2.84 or -1.17 (3 s.f.)

SECTION 8 : QUADRATIC GRAPH SKETCHING

Practice 1

Sketch the graph of $y = x^2 - 4x + 3$, stating clearing all intercepts and the turning point.

Practice 1 : $y = x^2 - 4x + 3$

Five-Pointers' Checklist

Shape

$\cup$

y-intercept (when x=0)

$y = 0^2 - 4(0) + 3$
$y = 3$

x-intercept (when y=0)

$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$

$x - 3 = 0 \quad \text{or} \quad x - 1 = 0$
$x = 3 \quad \text{or} \quad x = 1$

Turning Point

At $x = \dfrac{1+3}{2}$ | $y = 2^2 - 4(2) + 3$
$x = 2$ | $y = -1 \quad \therefore (2, -1)$

Line of Symmetry

$x = 2$

$y = x^2 - 4x + 3$
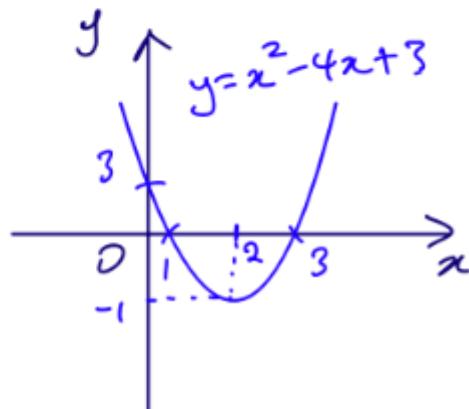
Practice 2

Sketch the graph of $y = x^2 - 2x + 24$, stating clearing all intercepts and the turning point.

Practice 2: $y = x^2 - 2x + 24$

Five-Pointers' Checklist

Shape

$\cup$

y-intercept (when x=0)

$y = 0^2 - 2(0) + 24$

$y = 24$

x-intercept (when y=0)

$x^2 - 2x + 24 = 0$

No x-intercepts as this cannot be solved

Turning Point

$x^2 - 2x + 24 = (x-1)^2 - (-1)^2 + 24$

$= (x-1)^2 - 1 + 24 \quad \therefore (1, 23)$

$= (x-1)^2 + 23$

Line of Symmetry

$x = 1$

$y = x^2 - 2x + 24$
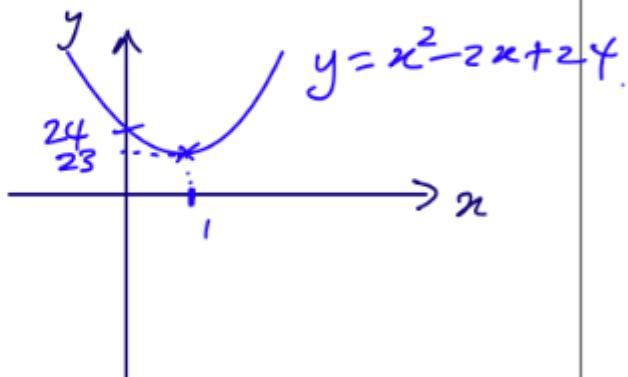
Practice 3

Sketch the graph of $y = (x - 5)(x + 5)$, stating clearing all intercepts and the turning point.

Practice 3: $y = (x-5)(x+5)$

Five-Pointers' Checklist

Shape

$\cup$

y-intercept (when x=0)

$y = (0-5)(0+5)$
$y = -25$

x-intercept (when y=0)

$(x-5)(x+5) = 0$
$x = 5 \quad \text{or} \quad x = -5$

Turning Point

At $x = \dfrac{5+(-5)}{2}$ | $y = -25$

$x = 0$

Line of Symmetry

$x = 0$
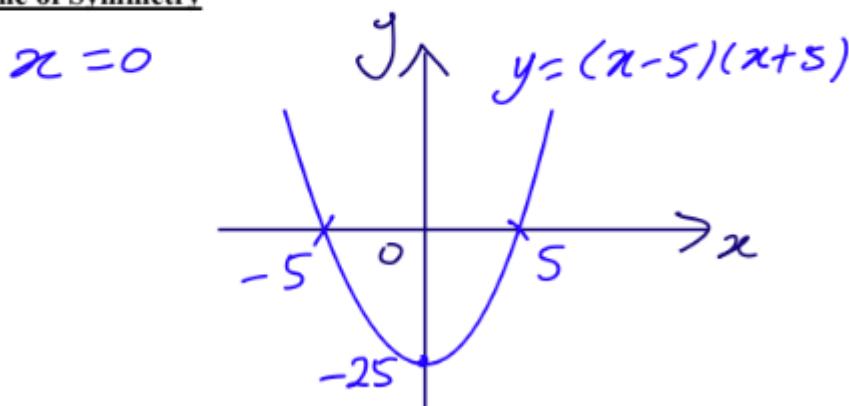
$y = (x-5)(x+5)$

Practice 4

Sketch the graph of $y = -(x - 5)(x + 5)$, stating clearing all intercepts and the turning point.

Practice 4: $y = -(x-5)(x+5)$

Five-Pointers' Checklist

Shape

y-intercept (when x=0)

$y = -(0-5)(0+5)$

$y = 25$

x-intercept (when y=0)

$-(x-5)(x+5) = 0$

$x = 5$ or $x = -5$

Turning Point

At $x = \dfrac{5+(-5)}{2}$

$x = 0 \longrightarrow y = 25$

Line of Symmetry

$x = 0$

$y = -(x-5)(x+5)$
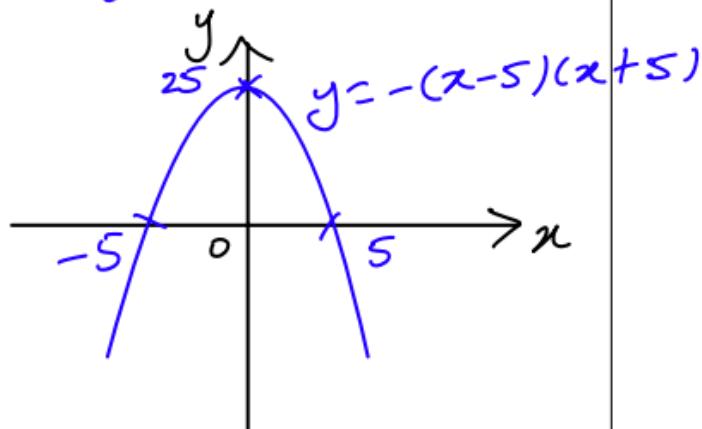
Practice 5

Sketch the graph of $y = x^2 - 2x + 1$, stating clearing all intercepts and the turning point.

Practice 5: $y = x^2 - 2x + 1$

Five-Pointers' Checklist

Shape

$\bigcup$

y-intercept (when x=0)

$y = 0^2 - 2(0) + 1$
$y = 1$

x-intercept (when y=0)

$x^2 - 2x + 1 = 0 \qquad x = 1$
$(x-1)^2 = 0$

Turning Point

At $x = \dfrac{1+1}{2}$

$x = 1 \longrightarrow y = 1^2 - 2(1) + 1 \qquad \therefore (1,0)$
$ y = 0$

Line of Symmetry

$x = 1$

$y = x^2 - 2x + 1$
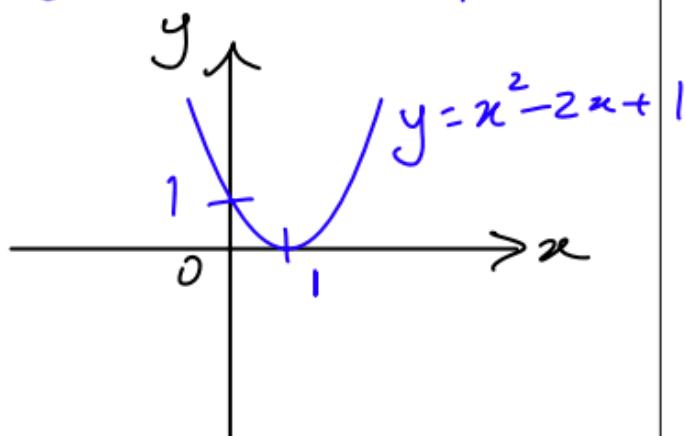
Practice 6

Sketch the graph of $y = -x^2 + 13x - 40$, stating clearing all intercepts and the turning point.

Practice 6: $y = -x^2 + 13x - 40$

Five-Pointers' Checklist

Shape

y-intercept (when x=0)

$y = -0^2 + 13(0) - 40$

$y = -40$

x-intercept (when y=0)

$-x^2 + 13x - 40 = 0$ | $-(x-5)(x-8) = 0$

$-(x^2 - 13x + 40) = 0$ | $x = 5$ or $x = 8$

Turning Point

At $x = \dfrac{5+8}{2}$ | $y = -6.5^2 + 13(6.5) - 40$

$x = 6.5$ | $y = 2.25$ ∴ $(6.5, 2.25)$

Line of Symmetry

$x = 6.5$
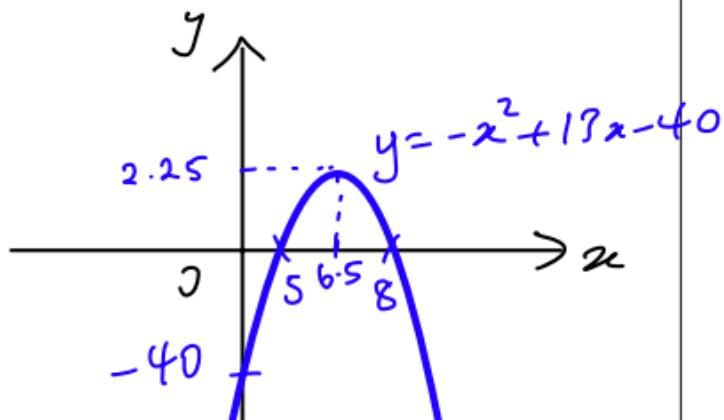
Practice 7

The following sketch is for the equation $y = (2x - a)(x + 5)$. Find the value of a.
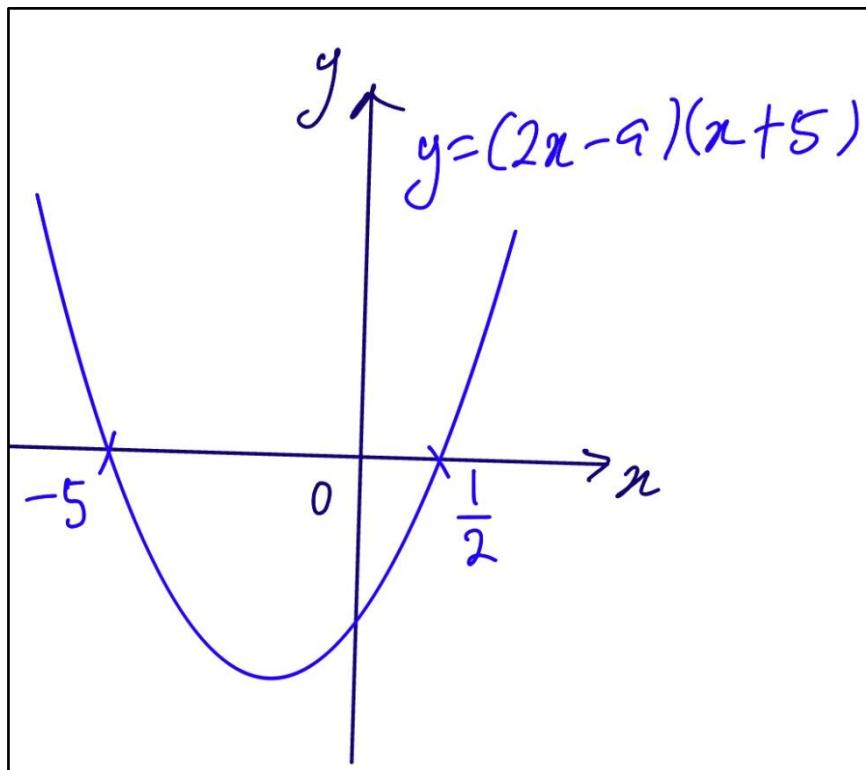
x-intercept occurs at *y = 0*

$(2x - a)(x + 5) = 0$

$(2x - a) = 0 \; or \; (x + 5) = 0$

$2x = a \; or \; x = -5$

$x = \dfrac{a}{2} \; or \; x = -5$

$x = \dfrac{a}{2} = \dfrac{1}{2}$

$a = 1$

Practice 8

Sketch the equation $y = (-x - 4)(x + 5)$.

Practice 8 : $y = (-x-4)(x+5)$

Five-Pointers' Checklist

Shape

y-intercept (when x=0)

$y = (-0-4)(0+5)$

$y = -20$

x-intercept (when y=0)

$(-x-4)(x+5) = 0$

$-x-4 = 0$ or $x+5 = 0$

$x = -4$ $x = -5$

Turning Point

At $x = \dfrac{-4+(-5)}{2}$

$x = -4.5 \longrightarrow y = (4.5-4)(-4.5+5)$

$= 0.25$ ∴ $(-4.5, 0.25)$

Line of Symmetry

$x = -4.5$

$y = (-x-4)(x+5)$
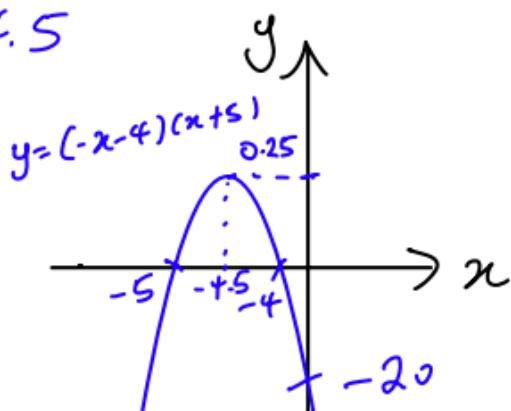
Practice 9

The quadratic equation $y = (-x - 4)(x + h)$ has a maximum point of $(-2.5 , 2.25)$.

Find the value of h.

x-intercept occurs at $y = 0$

$(-x - 4)(x + h) = 0$

$(-x - 4) = 0 \quad or \quad (x + h) = 0$

$x = -4 \quad or \quad x = -h$

$\dfrac{-4 + (-h)}{2} = -2.5$

$\dfrac{4 + h}{2} = 2.5$

$4 + h = 5$

$h = 5 - 4$

$h = 1$

Practice 10

The quadratic equation $y = ax^2 - 3x + c$ has y-intercept of -40
and passes through $(1, -42)$.

Find the values of a and of c.

At y-intercept, $x = 0$,

$-40 = c$

$c = -40$

At $(1, -42)$,

$-42 = a(1)^2 - 3(1) - 40$

$-42 = a - 43$

$-42 + 43 = a$

$a = 1$

REVIEW PRACTICE 2

(Completing the Square, Solving, Quadratic Sketching and Power Function Graphs)

Q1. By completing the square, solve $x^2 - 5x + 2 = 0$

$x^2 - 5x + 2 = 0$

$(x - \frac{5}{2})^2 - (-\frac{5}{2})^2 + 2 = 0$

$(x - 2.5)^2 - 6.25 + 2 = 0$

$(x - 2.5)^2 - 6.25 + 2 = 0$

$(x - 2.5)^2 - 4.25 = 0$

$(x - 2.5)^2 = 4.25$

$x - 2.5 = \pm\sqrt{4.25}$

$x = 2.5 \pm \sqrt{4.25}$

$x = 2.5 + \sqrt{4.25}$ or $x = 2.5 - \sqrt{4.25}$

$= 4.5615$ $= 0.4384$

≈ 4.56 ≈ 0.438 (3s.f.)

Q2. Sketch the graph of $y = (-x + 9)(x - 2)$, indicating clearly, if any, both axes' intercepts and turning point.

$$y = (-x+9)(x-2)$$

Five-Pointers' Checklist
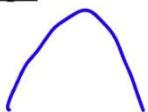
Shape

y-intercept (when x=0)

$$y = (-0+9)(0-2)$$
$$y = -18$$

x-intercept (when y=0)

$$(-x+9)(x-2) = 0$$
$$-x+9 = 0 \quad \text{or} \quad x-2 = 0$$
$$x = 9 \qquad\qquad x = 2$$

Turning Point

$$\text{At } x = \frac{9+2}{2}$$
$$x = 5.5 \Rightarrow y = (-5.5+9)(5.5-2)$$
$$y = 12.25$$

$$\therefore (5.5, 12.25)$$

Line of Symmetry

$$x = 5.5$$
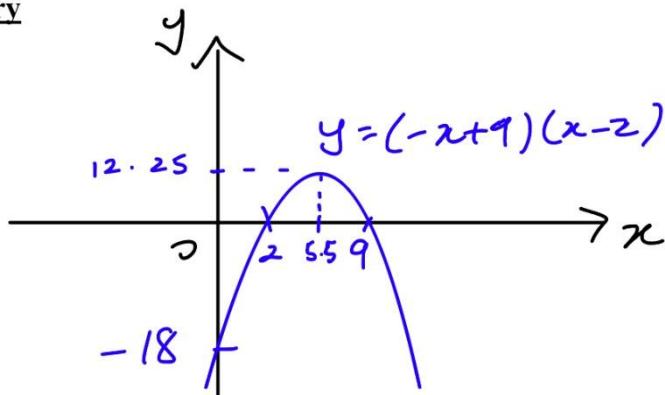
Q3. Solve the following algebraic expression

$$\frac{8}{x} - \frac{x}{8} = 0$$

$$\frac{8(8)}{8x} - \frac{x(x)}{8x} = 0$$

$$\frac{64 - x^2}{8x} = 0$$

$$64 - x^2 = 0$$

$$8^2 - x^2 = 0$$

$$(8 + x)(8 - x) = 0$$

$$(8 + x) = 0 \qquad \text{or} \qquad (8 - x) = 0$$

$$x = -8 \qquad \text{or} \qquad x = 8$$

Q4. Solve $\frac{3x-5}{2} - \frac{5-x}{3} = 2$

$$\frac{3x - 5}{2} - \frac{5 - x}{3} = 2$$

$$\frac{(3x - 5)(3)}{2(3)} - \frac{(5 - x)(2)}{3(2)} = 2$$

$$\frac{9x - 15}{6} - \frac{10 - 2x}{6} = 2$$

$$\frac{9x - 15 - 10 + 2x}{6} = 2$$

$$\frac{11x - 25}{6} = 2$$

$$11x - 25 = 12$$

$$11x = 12 + 25$$

$$11x = 37$$

$$x = \frac{37}{11}$$

$$x = 3\frac{4}{11}$$

Q5. Sketch the graph of $y = \frac{3}{x}$
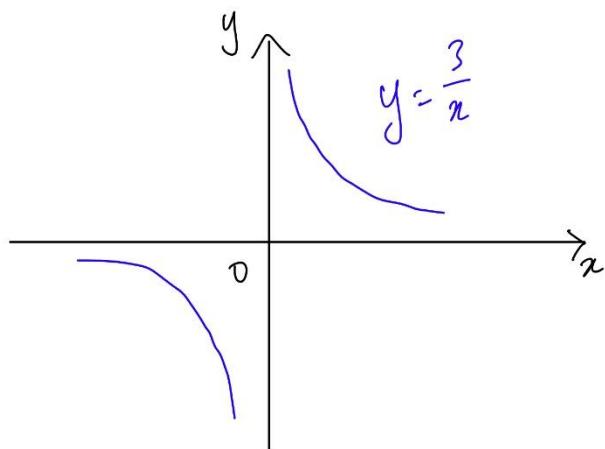
Q6. Given that $x^2 - 8x + 1 = (x + a)^2 - b$, find the values of a and of b.

$x^2 - 8x + 1$

$= (x - \frac{8}{2})^2 - (-\frac{8}{2})^2 + 1$

$= (x - 4)^2 - 16 + 1$

$= (x - 4)^2 - 15$

$a = -4, b = 15$

Q7. By completing the square, explain if $x^2 - 10x + 25$, is a perfect square.

$x^2 - 10x + 25$

$= (x - \frac{10}{2})^2 - (-\frac{10}{2})^2 + 25$

$= (x - 5)^2 - 25 + 25$

$= (x - 5)^2$

Considering that $x^2 - 10x + 25 = (x - 5)^2$, the equation is a perfect square.

Q8. Determine the value of a for the following quadratic graph.

$$a = \frac{4 + (-1)}{2}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$
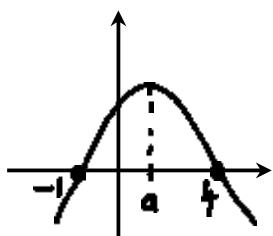
Q9. Solve $\frac{4}{2x+1} + \frac{5}{2x-1} = 2$

$$\frac{4}{2x+1} + \frac{5}{2x-1} = 2$$

$$\frac{4(2x-1)}{(2x+1)(2x-1)} + \frac{5(2x+1)}{(2x+1)(2x-1)} = 2$$

$$\frac{8x - 4 + 10x + 5}{(2x+1)(2x-1)} = 2$$

$$\frac{18x + 1}{(2x+1)(2x-1)} = 2$$

$$18x + 1 = 2(2x+1)(2x-1)$$

$$18x + 1 = 2(4x^2 - 1)$$

$$18x + 1 = 8x^2 - 2$$

$$0 = 8x^2 - 2 - 18x - 1$$

$$8x^2 - 18x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-18) \pm \sqrt{(-18)^2 - 4(8)(-3)}}{2(8)}$$

$$= \frac{18 \pm \sqrt{420}}{16}$$

$$= \frac{18 + \sqrt{420}}{16} \qquad \text{or} \qquad \frac{18 - \sqrt{420}}{16}$$

$$= \frac{18+\sqrt{420}}{16} \quad \text{or} \quad \frac{18-\sqrt{420}}{16}$$

$$= 2.4058 \quad \text{or} \quad = -0.1558$$

$$\approx 2.41 \quad \text{or} \quad \approx -0.156 \quad \text{(3 s.f.)}$$

Q10. Find the turning point of $y = x^2 - 4x - 21,$ and state if it's a maximum or minimum point.

$$x^2 - 4x - 21$$

$$= x^2 - 4x - 21$$

$$= (x - \frac{4}{2})^2 - (-\frac{4}{2})^2 - 21$$

$$= (x - 2)^2 - 4 - 21$$

$$= (x - 2)^2 - 25$$

Minimum Point = $(2, -25)$

SECTION 10 : INDICES

Practice 1

Simplify the following algebraic expressions, leaving your answer in positive index notation.

    (i) $a^2 b \times a^3 b^{-3}$

$a^2 b \times a^3 b^{-3}$

$$= a^{2+3} b^{1+(-3)}$$

$$= a^5 b^{-2}$$

$$= a^5 b^{-2}$$

$$= \frac{a^5}{b^2}$$

    (ii) $3p^2 q^{-3} \times 4p^{-4} q^{-1}$

$3p^2 q^{-3} \times 4p^{-4} q^{-1}$

$$= 12p^{2+(-4)} q^{-3+(-1)}$$

$$= 12p^{-2} q^{-4}$$

$$= \frac{12}{p^2 q^4}$$

(iii) $(a^3 b)^2 \div a^5 b^3$

$(a^3 b)^2 \div a^5 b^3$

$= a^6 b^2 \div a^5 b^3$

$= a^{6-5} b^{2-3}$

$= a^1 b^{-1}$

$= \dfrac{a}{b}$

(iv) $\left(\dfrac{5}{a}\right)^{-3}$

$\left(\dfrac{5}{a}\right)^{-3}$

$= \left(\dfrac{a}{5}\right)^3$

$= \dfrac{a^3}{5^3}$

$= \dfrac{a^3}{125}$

(v) $\left(\dfrac{x^{-2}}{y}\right)^3 \div \sqrt{\dfrac{x}{y}}$

$\left(\dfrac{x^{-2}}{y}\right)^3 \div \sqrt{\dfrac{x}{y}}$

$= \dfrac{x^{-6}}{y^3} \div \left(\dfrac{x}{y}\right)^{\frac{1}{2}}$

$= \dfrac{x^{-6}}{y^3} \div \dfrac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}$

$= \dfrac{x^{-6-\frac{1}{2}}}{y^{3-\frac{1}{2}}}$

$= \dfrac{x^{-6\frac{1}{2}}}{y^{2\frac{1}{2}}}$

$= \dfrac{1}{x^{6\frac{1}{2}} y^{2\frac{1}{2}}}$

Practice 2

Solve the following

(i) $3^k \div 81 = 3$

$3^k \div 3^4 = 3$

$3^{k-4} = 3^1$

$k - 4 = 1$

$k = 1 + 4$

$k = 5$

(ii) $9^{x-1} = 243(3^x)$

$(3^2)^{x-1} = 3^5(3^x)$

$3^{2x-2} = 3^{5+x}$

$2x - 2 = 5 + x$

$2x - x = 5 + 2$

$x = 7$

(iii) $\frac{1}{8} \div \sqrt{4^m} = 1$

$2^{-3} \div \sqrt{(2^2)^m} = 1$

$2^{-3} \div [(2^2)^m]^{\frac{1}{2}} = 2^0$

$2^{-3} \div 2^m = 2^0$

$2^{-3-m} = 2^0$

$-3 - m = 0$

$m = -3$

(iv) $\frac{10^{p^2}}{1000} \div \left(\frac{1}{10^{2p}}\right) = 1$

$\frac{10^{p^2}}{1000} \div \left(\frac{1}{10^{2p}}\right) = 1$

$\frac{10^{p^2}}{10^3} \div 10^{-2p} = 10^0$

$10^{p^2 - 3 - (-2p)} = 10^0$

$p^2 + 2p - 3 = 0$

$(p + 3)(p - 1) = 0$

$$(p + 3) = 0 \quad \text{or} \quad (p - 1) = 0$$

$$p = -3 \quad \text{or} \quad p = 1$$

(v) $\quad 3^p + 3^p + 3^p + 3^p = 36$

$$4(3^p) = 36$$

$$3^p = \frac{36}{4}$$

$$3^p = 9$$

$$3^p = 3^2$$

$$p = 2$$

SECTION 11 : SUBJECT OF FORMULA

Practice 1

Express *w* as the subject of the formula

$$-2w + 3 = r - w$$

$$-2w + 3 = r - w$$

$$3 - r = -w + 2w$$

$$3 - r = w$$

$$w = 3 - r$$

Practice 2

Express *x* as the subject of the formula

$$3(3 - x) = x(2 - 5b)$$

$$3(3 - x) = x(2 - 5b)$$

$$9 - 3x = 2x - 5bx$$

$$9 = 2x - 5bx + 3x$$

$$9 = 5x - 5bx$$

$$9 = x(5 - 5b)$$

$$x = \frac{9}{5 - 5b}$$

$$x = \frac{9}{5(1 - b)}$$

Practice 3

Make *v* the subject of the formula

$$3v^2 = x - 5v^2 + 8$$

$$3v^2 = x - 5v^2 + 8$$

$$3v^2 + 5v^2 = x + 8$$

$$8v^2 = x + 8$$

$$v^2 = \frac{x + 8}{8}$$

$$v = \pm \sqrt{\frac{x + 8}{8}}$$

Practice 4

Make *x* the subject of the formula

$$m + \sqrt{x} = n$$

$$\sqrt{x} = n - m$$

$$x = (n - m)^2$$

Practice 5

Express *p* as the subject of the formula

$$\sqrt{\frac{1 - p}{p}} = x$$

$$\sqrt{\frac{1 - p}{p}} = x$$

$$\frac{1 - p}{p} = x^2$$

$$1 - p = px^2$$

$$1 = px^2 + p$$

$$1 = p(x^2 + 1)$$

$$p = \frac{1}{x^2 + 1}$$

Practice 6

Make *u* the subject of the formula

$$v^2 = u^2 + 2as$$

$$v^2 = u^2 + 2as$$

$$v^2 - 2as = u^2$$

$$u^2 = v^2 - 2as$$

$$u = \pm\sqrt{v^2 - 2as}$$

Practice 7

Make *x* the subject of the formula

$$\frac{3 - x^2}{2 + bx^2} = k$$

$$\frac{3 - x^2}{2 + bx^2} = k$$

$$3 - x^2 = k(2 + bx^2)$$

$$3 - x^2 = 2k + bkx^2$$

$$3 - 2k = bkx^2 + x^2$$

$$3 - 2k = x^2(bk + 1)$$

$$x^2 = \frac{3 - 2k}{(bk + 1)}$$

$$x = \pm\sqrt{\frac{3 - 2k}{bk + 1}}$$

Practice 8

Express *a* in terms of *s*, *u* and *t*.

$$s = ut + \frac{1}{2}at^2$$

$$s = ut + \frac{1}{2}at^2$$

$$s - ut = \frac{1}{2}at^2$$

$$2(s - ut) = at^2$$

$$\frac{2(s - ut)}{t^2} = a$$

$$a = \frac{2(s - ut)}{t^2}$$

Practice 9

Make b the subject of the formula.

$$\frac{3 - 2b}{b + 9} = \frac{5}{8x}$$

$$\frac{3 - 2b}{b + 9} = \frac{5}{8x}$$

$$(3 - 2b)(8x) = 5(b + 9)$$

$$24x - 16bx = 5b + 45$$

$$24x - 45 = 5b + 16bx$$

$$24x - 45 = b(5 + 16x)$$

$$b = \frac{24x - 45}{5 + 16x}$$

Practice 10

Make x the subject of the formula

$$z = \sqrt{\frac{x}{x + y}}$$

$$z = \sqrt{\frac{x}{x + y}}$$

$$z^2 = \frac{x}{x + y}$$

$$z^2(x + y) = x$$

$$xz^2 + yz^2 = x$$

$$xz^2 - x = -yz^2$$

$$x(z^2 - 1) = -yz^2$$

$$x = \frac{-yz^2}{z^2 - 1}$$

SECTION 12 : LINEAR ALGEBRAIC INEQUALITY

Practice 1

Solve $5a - 2 > -2(a - 10)$

$5a - 2 > -2(a - 10)$

$5a - 2 > -2a + 20$

$5a + 2a > 20 + 2$

$7a > 22$

$a > \dfrac{22}{7}$

$a > 3\dfrac{1}{7}$

Practice 2

Write down the inequalities of *x* that satisfies this number line.

$-2 \leq x < 7$
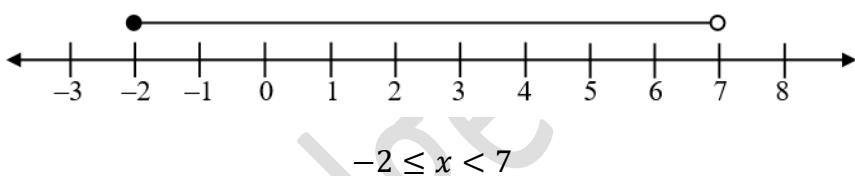
Practice 3

Solve $2(3 - 4x) < -(x - 9)$

$2(3 - 4x) < -(x - 9)$

$6 - 8x < -x + 9$

$6 - 9 < -x + 8x$

$-3 < 7x$

$7x > -3$

$x > -\dfrac{3}{7}$

Practice 4

Solve the inequalities $-6 < 3 - 2x \leq 9$ and write down all prime numbers that is satisfied by the inequality.

$-6 < 3 - 2x$	and	$3 - 2x \leq 9$
$-6 - 3 < -2x$		$-2x \leq 9 - 3$
$-2x > -9$		$-2x \leq 6$
$9 > 2x$		$-6 \leq 2x$
$2x < 9$		$2x \geq -6$
$x < 4.5$		$x \geq -3$

Therefore $-3 \leq x < 4.5$
Prime numbers $= 2$ and 3

Practice 5

Solve $1 + \frac{4-8y}{3} \geq 5(y - 5)$

$$1 + \frac{4 - 8y}{3} \geq 5(y - 5)$$

$$3 + 4 - 8y \geq 15(y - 5)$$

$$7 - 8y \geq 15y - 75$$

$$7 + 75 \geq 15y + 8y$$

$$82 \geq 23y$$

$$23y \leq 82$$

$$23y \leq 82$$

$$y \leq \frac{82}{23}$$

$$y \leq 3\frac{13}{23}$$

Practice 6

Solve $\quad 2 + \frac{-8a+2}{3} \leq -(4a+1)$

$2 + \dfrac{-8a+2}{3} \leq -(4a+1)$

$6 - 8a + 2 \leq -3(4a+1)$

$8 - 8a \leq -12a - 3$

$-8a + 12a \leq -3 - 8$

$4a \leq -11$

$a \leq -\dfrac{11}{4}$

$a \leq -2\dfrac{3}{4}$

Practice 7

Solve the inequalities $\frac{7}{3} < \frac{x}{2} - \frac{x}{3} + 2 \leq 3$ and represent the solution on a number line.

$\frac{7}{3} < \frac{x}{2} - \frac{x}{3} + 2$ $\qquad$ and $\qquad$ $\frac{x}{2} - \frac{x}{3} + 2 \leq 3$

$\qquad\qquad\qquad\qquad\qquad\qquad 3x - 2x + 12 \leq 18$

$14 < 3x - 2x + 12$ $\qquad\qquad\quad x + 12 \leq 18$

$14 < x + 12$ $\qquad\qquad\qquad\quad x \leq 18 - 12$

$14 - 12 < x$ $\qquad\qquad\qquad\quad x \leq 6$

$x > 2$

Therefore $2 < x \leq 6$
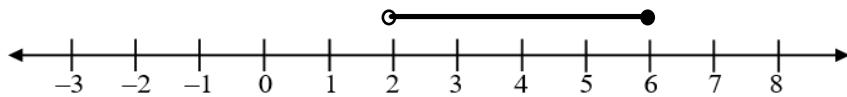
Practice 8

Solve the inequalities $8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$

$8 + x < 10 + \frac{3}{2}x$ 　　　　 and 　　　　 $\frac{3}{2}x + 2x \leq 15.5 - 10$

$8 - 10 < \frac{3}{2}x - x$ 　　　　　　　　　　　 $3.5x \leq 5.5$

$-2 < \frac{1}{2}x$ 　　　　　　　　　　　　　　 $x \leq \frac{5.5}{3.5}$

$\frac{1}{2}x > -2$ 　　　　　　　　　　　　　　 $x \leq \frac{11}{7}$

$x > -4$ 　　　　　　　　　　　　　　　　 $x \leq 1\frac{4}{7}$

Therefore $-4 < x \leq 1\frac{4}{7}$

Practice 9

Solve the following inequality $6 < 2x + \frac{3x-5}{4} \leq 4x + 2$

$6 < 2x + \frac{3x-5}{4}$ 　　 and 　　 $2x + \frac{3x-5}{4} \leq 4x + 2$

　　　　　　　　　　　　　　　 $8x + 3x - 5 \leq 16x + 8$

$24 < 8x + 3x - 5$ 　　　　　　 $-5 - 8 \leq 16x - 11x$

$24 + 5 < 11x$ 　　　　　　　 $-13 \leq 5x$

$29 < 11x$ 　　　　　　　　　 $5x \geq -13$

$11x > 29$ 　　　　　　　　　 $x \geq -\frac{13}{5}$

$x > \frac{29}{11}$ 　　　　　　　　　　 $x \geq -2\frac{3}{5}$

$x > 2\frac{7}{11}$

Therefore $x > 2\frac{7}{11}$

Practice 10

Solve the inequality $2x - 1 < 3x \leq \frac{2+3x}{3}$

$2x - 1 < 3x$	and	$9x \leq 2 + 3x$
$-1 < 3x - 2x$		$9x - 3x \leq 2$
$-1 < x$		$6x \leq 2$
$x > -1$		$x \leq \frac{1}{3}$

Therefore $-1 < x \leq \frac{1}{3}$

SECTION 13 : SIMULTANEOUS EQUATIONS

Practice 1

Solve the following sets of simultaneous equations

(a) $x + y = 3$……Eqn.1
   $x - y = -1$……Eqn.2
   Let's practise Elimination Method
   Eliminating y, Eqn.1 + Eqn.2
   $x + y + x - y = 3 + -1$
   $2x = 2$
   $x = 1$

   Insert $x = 1$ into Eqn.1
   $1 + y = 3$
   $y = 3 - 1$
     $= 2$

(b) $x + y = 5$……Eqn.1
   $x - y = 1$……Eqn.2
   Let's practise Substitution Method
   From Eqn.1, $y = 5 - x$

   Substitute $y = 5 - x$ into Eqn.2
   $x - (5 - x) = 1$
   $x - 5 + x = 1$
   $2x - 5 = 1$
   $2x = 1 + 5$
   $2x = 6$
   $x = \frac{6}{2}$
   $x = 3$
   $y = 5 - 3$
     $= 2$

(c) $3p + 8q = 27$……Eqn.1
$2p + 4q = 14$……Eqn.2
Let's practise Elimination Method
Eliminating p, we make both equations to $6p$
Eqn.1 × 2 : $6p + 16q = 54$……Eqn.3
Eqn.2 × 3 : $6p + 12q = 42$……Eqn.4

Eqn.3 – Eqn.4
$6p + 16q - (6p + 12q) = 54 - 42$
$4q = 12$
$q = \dfrac{12}{4}$
$= 3$

Insert $q = 3$ into Eqn.1
$3p + 8(3) = 27$
$3p + 24 = 27$
$3p = 27 - 24$
$3p = 3$
$p = \dfrac{3}{3}$
$= 1$

(d) $2a + 3c = 16$……Eqn.1
$a + c = 7$……Eqn.2
Let's practise Substitution Method
From Eqn.2, $c = 7 - a$

Substitute $c = 7 - a$ into Eqn.1
$2a + 3(7 - a) = 16$
$2a + 21 - 3a = 16$
$-a = 16 - 21$
$-a = -5$
$a = 5$
$c = 7 - 5$
$= 2$

(e) $3x - 2y = -9$……Eqn.1
$x = 4 - 4y$……Eqn.2
Let's practise Elimination Method
Eliminating x, we make both equations to $3x$

Eqn.2 × 3 : $3x = 12 - 12y$ ……Eqn.3

Eqn.3 – Eqn.1
$3x - (3x - 2y) = 12 - 12y - (-9)$
$2y = 21 - 12y$
$2y + 12y = 21$
$14y = 21$

$$y = \frac{21}{14}$$
$$= \frac{3}{2}$$
$$= 1\frac{1}{2}$$

Insert $y = 1\frac{1}{2}$ into Eqn.2

$$x = 4 - 4(1\frac{1}{2})$$
$$= 4 - 6$$
$$= -2$$

(f) $2y = x - 8$……Eqn.1
$3x - 4y = 19$……Eqn.2
Let's practise Substitution Method
From Eqn.1, $x = 2y + 8$

Substitute $x = 2y + 8$ into Eqn.2
$3(2y + 8) - 4y = 19$
$6y + 24 - 4y = 19$
$2y = 19 - 24$
$2y = -5$
$y = -\frac{5}{2}$
$y = -2.5$
$x = 2(-2.5) + 8$
  $= -5 + 8$
  $= 3$

(g) $3m + 2n = 7$……Eqn.1
$m = 5 - 4n$……Eqn.2
Let's practise Substitution Method
Substitute Eqn 2 into Eqn 1

$3(5 - 4n) + 2n = 7$
$15 - 12n + 2n = 7$
$-10n = 7 - 15$
$-10n = -8$
$n = \frac{-8}{-10}$
$n = \frac{4}{5}$
$m = 5 - 4 \times \frac{4}{5}$
$m = 1\frac{4}{5}$

(h) $\frac{y-4}{2} = x$……Eqn.1

$\frac{y}{x} = 3$ ……Eqn.2

Let's practise Substitution Method

From Eqn.1, $x = \frac{y-4}{2}$

Substitute $x = \frac{y-4}{2}$ into Eqn.2

$\frac{y}{\frac{y-4}{2}} = 3$

$y \div \frac{y-4}{2} = 3$

$y \times \frac{2}{y-4} = 3$

$\frac{2y}{y-4} = 3$

$2y = 3(y-4)$

$2y = 3y - 12$

$12 = 3y - 2y$

$12 = y$

$y = 12$

$x = \frac{12-4}{2}$

$= \frac{8}{2}$

$= 4$

(i) $\frac{2}{x} + \frac{1}{y} = 25$……Eqn.1

$\frac{1}{x} - \frac{1}{y} = 5$……Eqn.2

Let's practise Elimination Method
(Substitution will be tedious for this one. Easier to use Elimination as we observe a common term i.e. $\frac{1}{y}$ that can be eliminated from both equations)

Eliminating $\frac{1}{y}$ from both equations

Eqn.1 + Eqn.2

$$\frac{2}{x} + \frac{1}{y} + \frac{1}{x} - \frac{1}{y} = 25 + 5$$

$$\frac{3}{x} = 30$$

$$\frac{3}{30} = x$$

$$x = \frac{1}{10}$$

Insert $x = \frac{1}{10}$ into Eqn.1

$$\frac{2}{\left(\frac{1}{10}\right)} + \frac{1}{y} = 25$$

$$20 + \frac{1}{y} = 25$$

$$\frac{1}{y} = 25 - 20$$

$$\frac{1}{y} = 5$$

$$y = \frac{1}{5}$$

Practice 2

Jona goes to FOFULAR to buy 2 pencils and 3 erasers at a total cost of $3.50.

Dewi goes to the same FOFULAR to buy 3 pencils and 5 erasers at $5.50.

Find the cost of 1 pencil.

Find the cost of 1 eraser.

Let the cost of each pencil be p

Let the cost of each eraser be e

$2p + 3e = 3.50$……Eqn.1

$3p + 5e = 5.50$……Eqn.2

Let's practise Elimination Method

Eliminating p, we make both equations to $6p$.

Eqn.1 $\times$ 3 : $6p + 9e = 10.50$……Eqn.3

Eqn.2 $\times$ 2 : $6p + 10e = 11$……Eqn.4

Eqn.4 $-$ Eqn.3

$6p + 10e - (6p + 9e) = 11 - 10.50$

$e = 11 - 10.50$

= 0.50

Insert $e = 0.50$ into Eqn.1

$2p + 3(0.50) = 3.50$

$2p + 1.50 = 3.50$

$2p = 3.50 - 1.50$

$2p = 2$

$p = 1$

1 pencil costs $1.00

1 eraser costs $0.50

SECTION 14 : WORD PROBLEMS

Practice 1

Rajesh bought *m* kiwi fruits for $87. Find an expression, in terms of *m*, for the cost of one kiwi fruit in cents.

m kiwi= 8700 cents

1 kiwi $= \frac{8700}{m}$

Practice 2

For a Volunteer trip, some teachers and students walked 650 m from the school to a bus stop (so as to board a bus) at an average speed of *x* km/h. After the activity, they returned by the same route and walked from the same bus stop back to school, but at an average speed of 4 km/h slower than the earlier trip.

Express, in terms of *x*, the time in hours that the team took for the return trip.

time $= \frac{distance}{speed}$

$= \frac{0.650 km}{(x-4)\frac{km}{h}}$

$= \frac{0.65}{x-4} h$

Practice 3

Ali and Ben jogged for a full marathon

Ali's speed was *x* km/h

Ben's speed was 2km/h faster than Ali.

Write down an expression for

(a) Ali's time taken

(b) Ben's time taken

[A full marathon is 42 km]

(a) Ali's time taken

$$\text{time} = \frac{distance}{speed}$$

$$= \frac{42 \ km}{x \ km/h}$$

$$= \frac{42}{x} h$$

(a) Ben's time taken

$$\text{time} = \frac{42 km}{(x+2) \ km/h}$$

$$= \frac{42}{x+2} h$$

Practice 4

A rectangle has a length of $(x + 3) \ m$ and an area of $(x^2 + x - 6) \ m^2$. Find an expression for the breadth of the rectangle and simplify.

$$\text{Breadth} = \frac{Area}{Length}$$

$$= \frac{x^2 + x - 6}{x + 3}$$

$$= \frac{(x+3)(x-2)}{x+3}$$

$$= (x - 2) m$$

Practice 5

The distance between two towns, Alpha and Beta is 60 km. Initially, Mr Singh travelled from Alpha to Beta at *x* km/h.

(a) Write down the time taken to travel from Alpha to Beta.

(b) On the return journey from Beta to Alpha, Mr Singh travelled 7 km/h slower. Find the time taken for the return journey.

(a) $$\text{time} = \frac{distance}{speed}$$

$$= \frac{60 \ km}{x \ km/h}$$

$$= \frac{60}{x} h$$

(b) $$\text{time} = \frac{distance}{speed}$$

$$= \frac{60 \ km}{(x-7) km/h}$$

$$=\frac{60}{x-7}\, h$$

Practice 6

A photo of area 96 cm^2 is placed on a picture frame of 18 cm by 14 cm with a border of uniform width as shown.
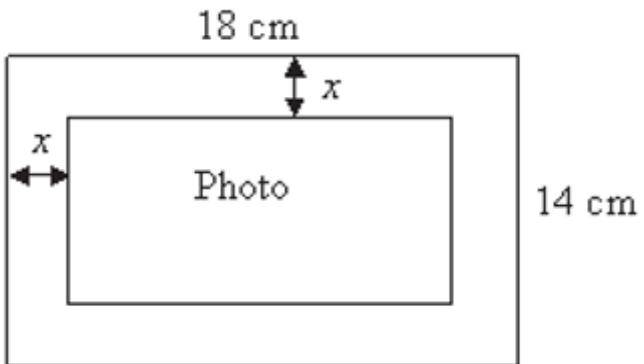
Form an equation in x and show that it reduces to $x^2 - 16x + 39 = 0$

Length of Photo $= 18 - 2x$

Width of Photo $= 14 - 2x$

$(18 - 2x)(14 - 2x) = 96$

$(18 - 2x)(14 - 2x) = 96$

$252 - 36x - 28x + 4x^2 = 96$

$252 - 36x - 28x + 4x^2 - 96 = 0$

$4x^2 - 64x + 156 = 0$

$x^2 - 16x + 39 = 0$ [shown]

Practice 7

Amanda ran the 21 km of a half-marathon race at an average speed of x km/h.

(a) Write down, in terms of x, an expression for the number of hours it took her to complete the race.

(b) Deborah ran the same race at an average speed which is 3km/h faster than Amanda's speed. Write down, in terms of x, an expression for the number of hours which Deborah took.

(c) Given that the difference between the two times was 20 minutes, write down an equation in x and show that it reduces to $x^2 + 3x - 189 = 0$

(a) time $= \dfrac{distance}{speed}$

$= \dfrac{21km}{x\ km/h}$

$= \dfrac{21}{x}h$

(b) time $= \dfrac{distance}{speed}$

$= \dfrac{21km}{(x+3)\ km/h}$

$= \dfrac{21}{x+3}h$

(c) longer time → $Amanda$

   shorter time → $Deborah$

   longer time – shorter time $= \dfrac{20}{60}h$

   longer time – shorter time $= \dfrac{1}{3}h$

$\dfrac{21}{x} - \dfrac{21}{x+3} = \dfrac{1}{3}$

$\dfrac{21(x+3)}{x(x+3)} - \dfrac{21(x)}{x(x+3)} = \dfrac{1}{3}$

$\dfrac{21x + 63 - 21x}{x(x+3)} = \dfrac{1}{3}$

$\dfrac{63}{x(x+3)} = \dfrac{1}{3}$

$63(3) = x(x+3)$

$189 = x^2 + 3x$

$0 = x^2 + 3x - 189$

$x^2 + 3x - 189 = 0$ [shown]

Practice 8

A tank has a capacity of 1080 litres.

(a) Tap A fills the tank at a rate of x litres per minute.

Write an expression, in terms of x, the time taken in minutes, by Tap A to fill up the tank completely.

(b) Tap B fills the tank at a rate of 2 litres per minute slower than Tap A.

 Write an expression, in terms of x, the time taken in minutes, by Tap B to fill up the tank completely.

(c)

The difference in time taken by Tap A and Tap B to fill the tank completely is 40 minutes and 36 seconds. Write down an equation in x to represent this information and show that it reduces to $203x^2 - 406x - 10800 = 0$

(a)

$$\text{time} = \frac{1080\ l}{x\ l/min}$$

$$= \frac{1080}{x}\ min$$

(b)

$$\text{time} = \frac{1080\ l}{(x-2)\ l/min}$$

$$= \frac{1080}{x-2}\ min$$

(c)

longer time$\rightarrow tap\ B$

shorter time$\rightarrow tap\ A$

$$\text{longer time – shorter time} = \left(40 + \frac{36}{60}\right) minutes$$

$$= 40\frac{3}{5} minutes$$

$$\text{longer time – shorter time} = 40\frac{3}{5}\ minutes$$

$$\frac{1080}{x-2} - \frac{1080}{x} = 40\frac{3}{5}$$

$$\frac{1080(x)}{x(x-2)} - \frac{1080(x-2)}{x(x-2)} = 40\frac{3}{5}$$

$$\frac{1080(x)}{x(x-2)} - \frac{1080(x-2)}{x(x-2)} = \frac{203}{5}$$

$$\frac{1080x - 1080x + 2160}{x(x-2)} = \frac{203}{5}$$

$$\frac{2160}{x(x-2)} = \frac{203}{5}$$

$$2160(5) = 203x(x-2)$$

$$10800 = 203x^2 - 406x$$

$$0 = 203x^2 - 406x - 10800$$

$$203x^2 - 406x - 10800 = 0 \text{ [shown]}$$

Practice 9

PQRS is a trapezium which PS is parallel to *QR* and angle *PSR* = 90°

$PS = (5x - 6)$ cm , $QR = (3x - 8)$ cm and $SR = (2x - 1)$ cm
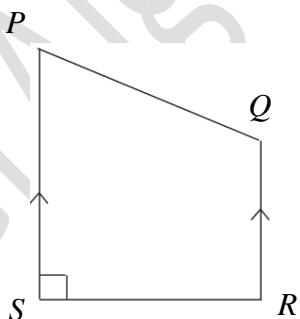
(a) Write down, in terms of *x*, an expression for the area of the trapezium.

(b) Given that the area of the trapezium is 33 cm^2, form an equation in *x*, and show that it reduces to $4x^2 - 9x - 13 = 0$

(c) Solve $4x^2 - 9x - 13 = 0$

(d) Hence find the length of *PS*.

(a)

Area of trapezium $= \frac{1}{2} \times (sum\ of\ parallel\ sides) \times height\ between\ parallel\ sides$

$$= \frac{1}{2} \times (5x - 6 + 3x - 8) \times (2x - 1)$$

$$= \frac{1}{2} \times (8x - 14) \times (2x - 1)$$

$$= (4x - 7) \times (2x - 1)$$

$$= 8x^2 - 4x - 14x + 7$$

$$= 8x^2 - 18x + 7$$

(b)

Area of trapezium $= 8x^2 - 18x + 7$

$8x^2 - 18x + 7 = 33$

$8x^2 - 18x + 7 - 33 = 0$

$8x^2 - 18x - 26 = 0$

$4x^2 - 9x - 13 = 0$ [shown]

(c)

$4x^2 - 9x - 13 = 0$

$(4x - 13)(x + 1) = 0$

$(4x - 13) = 0 \qquad$ or $\qquad (x + 1) = 0$

$4x = 13 \qquad$ or $\qquad x = -1$

$x = \frac{13}{4} \qquad$ or $\qquad x = -1$

$x = 3\frac{1}{4}$

(d) $x = -1$(rejected since length cannot be negative)

$PS = 5x - 6$

$$= 5\left(3\frac{1}{4}\right) - 6$$

$$= 10\frac{1}{4} \text{ cm}$$

Practice 10

Mr. Teo bought x thumbdrives for $1050 and intended to sell them at a profit of $6 per thumbdrive.

(a) Write down an expression in terms of x, the selling price of each thumbdrives.

(b) He received $1050 from selling the thumbdrives with 20 left unsold. Form an equation in terms of x and show that it reduces to $x^2 - 20x - 3500 = 0$.

(c) (i) Solve the equation $x^2 - 20x - 3500 = 0$ to find the number of thumbdrives he bought.

(ii) Hence, find the selling price of each thumbdrive.

(a)

Cost price of each thumbdrive= $\dfrac{1050}{x}$

Selling price of each thumbdrive = $(\dfrac{1050}{x} + 6)$

(b) Number of thumbdrives sold = $(x - 20)$

$\quad$ Sales = $(x - 20) (\dfrac{1050}{x} + 6)$

$\quad$ $1050 = (x - 20) (\dfrac{1050}{x} + 6)$

$\quad$ $1050x = (x - 20) (\dfrac{1050}{x} + 6) (x)$

$\quad$ $1050x = (x - 20) (1050 + 6x)$

$\quad$ $(x - 20)(1050 + 6x) = 1050x$

$\quad$ $1050x + 6x^2 - 21000 - 120x = 1050x$

$\quad$ $6x^2 - 120x - 21000 = 0$

$\quad$ $x^2 - 20x - 3500 = 0$ [shown]

(c) (i)
$x^2 - 20x - 3500 = 0$

$(x + 50)(x - 70) = 0$

$\quad (x + 50) = 0 \qquad$ or $\qquad (x - 70) = 0$

$\quad x = -50$ (rejected) $\quad$ or $\qquad\qquad x = 70$

$\quad$ Number of thumbdrives bought = 70

(ii) Selling price of each thumbdrive = $(\dfrac{1050}{70} + 6)$

$$= \$21$$

REVIEW PRACTICE 3

(Word Problems, Simultaneous Equations, Inequality, Subject of Formula and Indices)

Q1. Given that $1 - \frac{a-b}{b+2c} = \frac{2a-1}{1}$, make b the subject of the formula.

$$1 - \frac{a - b}{b + 2c} = \frac{2a - 1}{1}$$

$$\frac{b + 2c}{b + 2c} - \frac{a - b}{b + 2c} = \frac{2a - 1}{1}$$

$$\frac{b + 2c - a + b}{b + 2c} = \frac{2a - 1}{1}$$

$$\frac{2b + 2c - a}{b + 2c} = \frac{2a - 1}{1}$$

$$2b + 2c - a = (b + 2c)(2a - 1)$$

$$2b + 2c - a = 2ab - b + 4ac - 2c$$

$$2b + b - 2ab = 4ac - 2c - 2c + a$$

$$3b - 2ab = 4ac - 4c + a$$

$$b(3 - 2a) = 4ac - 4c + a$$

$$b = \frac{4ac - 4c + a}{3 - 2a}$$

Q2. Solve the inequality $-5 \le \frac{4x-3}{2} < 3$.

$-5 \le \frac{4x-3}{2}$ and	$\frac{4x-3}{2} < 3$
$-10 \le 4x - 3$	$4x - 3 < 6$
$-10 + 3 \le 4x$	$4x < 6 + 3$
$-7 \le 4x$	$4x < 9$
$4x \ge -7$	$x < \frac{9}{4}$
$x \ge -\frac{7}{4}$	$x < 2\frac{1}{4}$
$x \ge -1\frac{3}{4}$	

Therefore $-1\frac{3}{4} \le x < 2\frac{1}{4}$

Q3. Solve the simultaneous equations below giving your answer in exact values.

$1.5x - y = 3.5$…….Eqn.1

$6x + 1\frac{2}{3}y = 7$…….Eqn.2

From Eqn.1, $y = 1.5x - 3.5$

Substitute $y = 1.5x - 3.5$ into Eqn.2

$6x + 1\frac{2}{3}(1.5x - 3.5) = 7$

$6x + 1\frac{2}{3}(1.5x - 3.5) = 7$

$6x + 2.5x - 5\frac{5}{6} = 7$

$8.5x - 5\frac{5}{6} - 7 = 0$

$8.5x - 12\frac{5}{6} = 0$

$8.5x = 12\frac{5}{6}$

$x = 1\frac{26}{51}$

$y = 1.5(1\frac{26}{51}) - 3.5$

$\quad = -1\frac{4}{17}$

Q4. Solve the following equation

$\dfrac{\sqrt{5^{4x}}}{25} = 1$

$\dfrac{(5^{4x})^{\frac{1}{2}}}{5^2} = 5^0$

$\dfrac{5^{2x}}{5^2} = 5^0$

$5^{2x-2} = 5^0$

$2x - 2 = 0$

$2x = 2$

$x = \dfrac{2}{2}$

$x = 1$

Q5. Express $\frac{3a}{b^2} \div \sqrt{\frac{a^3}{b}}$ in positive index notation.

$$\frac{3a}{b^2} \div (\frac{a^3}{b})^{\frac{1}{2}}$$

$$= \frac{3a}{b^2} \div \frac{a^{1.5}}{b^{0.5}}$$

$$= \frac{3a^{1-1.5}}{b^{2-0.5}}$$

$$= \frac{3a^{-0.5}}{b^{1.5}}$$

$$= \frac{3}{a^{0.5}b^{1.5}}$$

Q6. Alex bought some longans for \$360. He paid \$$x$ for each kilogram of longans.

(a) Write down an expression, in terms of x, for the number of kilogram of longans that he bought.

During the delivery, 5 kilogram of his longans were squashed. He sold the remainder of the longans at 60 cents more per kilogram than he paid for.

(b) Write down, in terms of x, for the sum of money he received for the remaining longans.

(c) He made a profit of \$171.

Write down an equation in x to represent this information and show that it reduces to $5x^2 + 174x - 216 = 0$

(d) Solve the equation and hence find the price that he paid for each kilogram of longans.

(a)

Number of Kilograms of Longans Bought→ $\frac{360}{x}$

(b)

Number of Kilograms of Longans Remaining→ $\frac{360}{x} - 5$

Sales → $(\frac{360}{x} - 5)(x + 0.60)$

(c)

Sales – Cost = Profit

$$\left(\frac{360}{x} - 5\right)(x + 0.60) - 360 = 171$$

$$\left(\frac{360}{x} - 5\right)(x + 0.60) = 171 + 360$$

$$360 + \frac{216}{x} - 5x - 3 = 171 + 360$$

$$360x + 216 - 5x^2 - 3x = 171x + 360x$$

$$-5x^2 - 3x - 171x - 360x + 360x + 216 = 0$$

$$-5x^2 - 174x + 216 = 0$$

$$5x^2 + 174x - 216 = 0 \text{ [shown]}$$

(d)

$$(5x - 6)(x + 36) = 0$$

$$(5x - 6) = 0 \quad \text{or} \quad (x + 36) = 0$$

$$5x = 6 \quad \text{or} \quad (x + 36) = 0$$

$$5x = 6 \quad \text{or} \quad x = -36$$

$$x = \frac{6}{5} \quad \text{or} \quad x = -36$$

$$x = 1.2 \quad \text{or} \quad x = -36 \text{ (rejected as price cannot be negative)}$$

He paid \$1.20 for each kilogram of longan.

Q7. Solve for m

$$2^m + 2^m + 2^m = 24$$

$$3(2^m) = 3(8)$$

$$2^m = 2^3$$

$$m = 3$$

Q8. Solve the inequality $3x - 1 \leq 2x + 9 < 7x + 4$

$3x - 1 \leq 2x + 9$	and	$2x + 9 < 7x + 4$
$3x - 2x \leq 9 + 1$		$9 - 4 < 7x - 2x$
$x \leq 10$		$5 < 5x$
$x \leq 10$		$1 < x$

Therefore $1 < x \leq 10$

Q9. Simplify $\frac{5p^2}{6q^3} \div \frac{3p^2}{8q^4}$

$$\frac{\frac{5}{3}p^{2-2}}{\frac{6}{8}q^{3-4}}$$

$$= \frac{\frac{5}{3}}{\frac{6}{8}q^{-1}}$$

$$= \frac{\frac{5}{3}}{\frac{6}{8}q^{-1}}$$

$$= \frac{\frac{5}{3}q}{\frac{6}{8}}$$

$$= \frac{5}{3}q \div \frac{6}{8}$$

$$= \frac{5}{3}q \times \frac{8}{6}$$

$$= \frac{40}{18}q$$

$$= 2\frac{2}{9}q$$

Q10. Solve the simultaneous equations below giving your answer in exact values.

$\frac{3}{x} - \frac{2}{y} = -14$……Eqn.1

$\frac{1}{x} - \frac{1}{y} = -8$……Eqn.2

Eliminating $\frac{1}{y}$, we make both equations to $\frac{2}{y}$

Eqn.2 × 2 : $\frac{2}{x} - \frac{2}{y} = -16$……Eqn.3

Eqn.3 <mark>-</mark> Eqn.1

$$\frac{2}{x} - \frac{2}{y} - \left(\frac{3}{x} - \frac{2}{y}\right) = -16 - (-14)$$

$$-\frac{1}{x} = -2$$

$$x = \frac{1}{2}$$

Insert $x = \frac{1}{2}$ into Eqn. 2

$$\frac{1}{\frac{1}{2}} - \frac{1}{y} = -8$$

$$2 - \frac{1}{y} = -8$$

$$\frac{1}{y} = 2 + 8$$

$$\frac{1}{y} = 10$$

$$y = \frac{1}{10}$$

ALGEBRA TEST

Total Duration : 1 hour 15 minutes

Total Marks : 50 marks

Q1. Factorise $(x - 7)^3 - 4x + 28$ completely [3]

$(x - 7)^3 - 4x + 28$

$= (x - 7)^3 - 4(x - 7)$

$= (x - 7)[(x - 7)^2 - 4]$

$= (x - 7)[(x - 7)^2 - 2^2]$

$= (x - 7)(x - 7 + 2)(x - 7 - 2)$

$= (x - 7)(x - 5)(x - 9)$

Q2. Given that $\sqrt{\frac{r}{7+q^2}} - p = r$

(i) evaluate p when $q = -3$ and $r = 36$ [1]

(ii) express q in terms of p and r [3]

(i)

$$\sqrt{\frac{r}{7+q^2}} - p = r$$

$$p = \sqrt{\frac{r}{7+q^2}} - r$$

$$p = \sqrt{\frac{36}{7+(-3)^2}} - 36$$

$$p = \sqrt{\frac{36}{16}} - 36$$

$$p = \frac{6}{4} - 36$$

$$p = -34.5$$

(ii) $\sqrt{\frac{r}{7+q^2}} = r + p$

$$\frac{r}{7+q^2} = (r+p)^2$$

$$7 + q^2 = \frac{r}{(r+p)^2}$$

$$q^2 = \frac{r}{(r+p)^2} - 7$$

$$q^2 = \frac{r}{(r+p)^2} - \frac{7(r+p)^2}{(r+p)^2}$$

$$q^2 = \frac{r - 7(r+p)^2}{(r+p)^2}$$

$$q = \pm\sqrt{\frac{r - 7(r+p)^2}{(r+p)^2}}$$

Q3. Solve the following equation   [3]

$$\frac{\sqrt{3^{2x}}}{81} = \frac{1}{\sqrt[3]{9}}$$

$$\frac{(3^{2x})^{\frac{1}{2}}}{3^4} = \frac{3^0}{(3^2)^{\frac{1}{3}}}$$

$$\frac{(3^{2x})^{\frac{1}{2}}}{3^4} = \frac{3^0}{(3^2)^{\frac{1}{3}}}$$

$$\frac{3^x}{3^4} = \frac{3^0}{3^{\frac{2}{3}}}$$

$$3^{x-4} = 3^{0-\frac{2}{3}}$$

$$x - 4 = -\frac{2}{3}$$

$$x = -\frac{2}{3} + 4$$

$$x = 3\frac{1}{3}$$

Q4.

Julia works in a coffee café that pays her a wage of \$$x$ per hour. In January, her salary was \$2000.

(a) Write down an expression in terms of x, for the number of hours she worked in January.

[1]

(b) From February onwards, Julia's wage was increased by \$1.50 per hour. If she also received \$2000 in February, write down an expression in terms of x, for the number of hours she worked in February.

[1]

(c) If Julia worked 13 hours less in February than in January, form an equation in x and show that it reduces to $26x^2 + 39x - 6000 = 0$

[3]

(d) Solve the equation $26x^2 + 39x - 6000 = 0$, giving both answers correct to two decimal places. [3]

(e) Calculate the minimum number of hours Julia needs to work in March if she aims to earn a salary of at least \$3000. [1]

(a)

Number of Hours worked in January $\rightarrow \dfrac{2000}{x}$ h

(b)

Number of Hours worked in February $\rightarrow \dfrac{2000}{(x+1.50)}$ h

(c)

$$\frac{2000}{x} - \frac{2000}{(x+1.50)} = 13$$

$$\frac{2000(x+1.50)}{x(x+1.50)} - \frac{2000(x)}{x(x+1.50)} = 13$$

$$\frac{2000x + 3000 - 2000x}{x(x+1.50)} = 13$$

$$\frac{3000}{x(x+1.50)} = 13$$

$$3000 = 13x(x+1.50)$$

$$0 = 13x^2 + 19.5x - 3000$$

$$13x^2 + 19.5x - 3000 = 0$$

$$26x^2 + 39x - 6000 = 0 \text{ [shown]}$$

(d)

$$26x^2 + 39x - 6000 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-39 \pm \sqrt{(39)^2 - 4(26)(-6000)}}{2(26)}$$

$$x = \frac{-39 \pm \sqrt{625521}}{52}$$

$x = \frac{-39 + \sqrt{625521}}{52}$ or $x = \frac{-39 - \sqrt{625521}}{52}$

$x = \frac{-39 + \sqrt{625521}}{52}$ or $x = \frac{-39 - \sqrt{625521}}{52}$

$x = 14.4595$ or $x = -15.9595$

$x = 14.46$ or $x = -15.96$ (2d.p.)

(e)

Hourly wage in March → $x + 1.50$

$$= 14.4595 + 1.50$$

$$= \$15.9595$$

Number of Hours in March→ $\frac{3000}{15.9595}$

$$= 187.9758 hours$$

$$\approx 188 hours$$

Q5. Solve this equation by completing the square   [3]

$$-6x^2 + 15x - 6 = 0$$

$$x^2 - \frac{15}{6}x + 1 = 0$$

$$x^2 - \frac{5}{2}x + 1 = 0$$

$$\left(x - \frac{5}{4}\right)^2 - (\frac{-5}{4})^2 + 1 = 0$$

$$\left(x - \frac{5}{4}\right)^2 - \frac{25}{16} + 1 = 0$$

$$\left(x - \frac{5}{4}\right)^2 - \frac{9}{16} = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{9}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{9}{16}}$$

$$x = \frac{5}{4} \pm \sqrt{\frac{9}{16}}$$

$$x = \frac{5}{4} + \sqrt{\frac{9}{16}} \qquad \text{or} \quad x = \frac{5}{4} - \sqrt{\frac{9}{16}}$$

$$x = 2 \qquad\qquad \text{or} \quad x = 0.5$$

Q6. Solve the inequalities $\frac{x}{2} - 5 \leq 3(x + 2) < 12$ and represent your solutions on a number line. [4]

|		
$\frac{x}{2} - 5 \leq 3(x + 2)$	and	$3(x + 2) < 12$
$x - 10 \leq 6(x + 2)$		$3x + 6 < 12$
$x - 10 \leq 6x + 12$		$3x < 12 - 6$
$-10 - 12 \leq 6x - x$		$3x < 6$
$-22 \leq 5x$		$x < 2$

$$5x \geq -22$$

$$x \geq -\frac{22}{5}$$

$$x \geq -4\frac{2}{5}$$

Therefore, $-4\frac{2}{5} \leq x < 2$

$$-4\frac{2}{5}$$
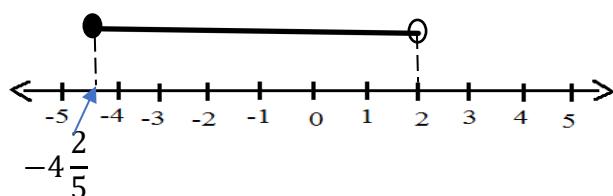
Q7. Solve the following equations.

(a) $12 - 3x^2 = 16x$ [2]

(b) $(2x - 1)^2 = 3x + 11$ [2]

(c) $15^{x+2} = 1$ [2]

(d) $\frac{x+2}{x^2-5x+6} - \frac{x}{4x-12} = \frac{13}{8}$ [4]

(a)

$-3x^2 - 16x + 12 = 0$

$(-3x + 2)(x + 6) = 0$

$(-3x + 2) = 0$ or $(x + 6) = 0$

$x = \frac{2}{3}$ or $x = -6$

(b)

$(2x)^2 - 2(2x)(1) + 1^2 - 3x - 11 = 0$

$4x^2 - 4x + 1 - 3x - 11 = 0$

$4x^2 - 7x - 10 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-7) \pm \sqrt{209}}{8}$

$x = \frac{7 \pm \sqrt{209}}{8}$

$x = \frac{7 + \sqrt{209}}{8}$ or $x = \frac{7 - \sqrt{209}}{8}$

$x = 2.6821$ or $x = -0.9321$

$x \approx 2.68$ or $x \approx -0.932$ (3 s.f.)

(c) $15^{x+2} = 15^0$

$x + 2 = 0$

$x = -2$

(d)

$$\frac{x+2}{(x-2)(x-3)} - \frac{x}{4(x-3)} = \frac{13}{8}$$

$$\frac{4(x+2)}{4(x-2)(x-3)} - \frac{x(x-2)}{4(x-3)(x-2)} = \frac{13}{8}$$

$$\frac{4(x+2) - x(x-2)}{4(x-2)(x-3)} = \frac{13}{8}$$

$$\frac{4x+8-x^2+2x}{4(x-2)(x-3)} = \frac{13}{8}$$

$$\frac{6x+8-x^2}{4(x-2)(x-3)} = \frac{13}{8}$$

$$8(6x+8-x^2) = 52(x-2)(x-3)$$

$$48x+64-8x^2 = 52(x^2-5x+6)$$

$$48x+64-8x^2 = 52x^2-260x+312$$

$$0 = 52x^2+8x^2-260x-48x+312-64$$

$$60x^2-308x+248 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-308) \pm \sqrt{(-308)^2-4(60)(248)}}{2(60)}$$

$$= \frac{308 \pm \sqrt{35344}}{120}$$

$$= \frac{308+\sqrt{35344}}{120} \quad \text{or} \quad = \frac{308-\sqrt{35344}}{120}$$

$$= \frac{308+\sqrt{35344}}{120} \quad \text{or} \quad = \frac{308-\sqrt{35344}}{120}$$

$$= 4.1333 \qquad\qquad = 1$$

$$\approx 4.13 \; (3 \; s.f.)$$

Q8. Sketch the graph of $y = x^2 - 6x - 7$, indicating clearly any intercepts and the turning point. [2]
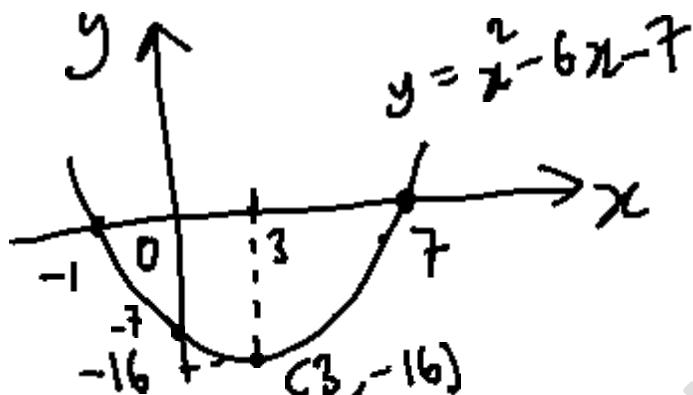
Q9. Expand and simplify $(x + 3)(4 - x) - 3(2 - 3x)^2$ [2]

$(x + 3)(4 - x) - 3(2 - 3x)^2$

$= (x + 3)(4 - x) - 3(2 - 3x)^2$

$= 4x + 12 - x^2 - 3x - 3[(2)^2 - 2(2)(3x) + (3x)^2]$

$= -x^2 + x + 12 - 3[4 - 12x + 9x^2]$

$= -x^2 + x + 12 - 12 + 36x - 27x^2$

$= -28x^2 + 37x$

Q10. Given that $\frac{z-x}{x-3z} = 5$, find the value of $\frac{z}{x}$ [2]

$$\frac{z - x}{x - 3z} = 5$$

Divide all left side terms by x

$$\frac{\frac{z}{x} - 1}{1 - 3\frac{z}{x}} = 5$$

$$\frac{z}{x} - 1 = 5(1 - 3\frac{z}{x})$$

$$\frac{z}{x} - 1 = 5 - 15\frac{z}{x}$$

$$\frac{z}{x} + 15\frac{z}{x} = 5 + 1$$

$$16\frac{z}{x} = 6$$

$$\frac{z}{x} = \frac{6}{16}$$

$$\frac{z}{x} = \frac{3}{8}$$

Q11. Simplify $\frac{2-r}{r-2}$ [2]

$$\frac{2-r}{r-2}$$

$$= \frac{-(r-2)}{r-2}$$

$$= -1$$

Q12. Express as a single fraction in its simplest form. [3]

(a) $\frac{3x}{2-3y} + \frac{6x}{9y^2-4}$

(b) $\frac{x+2}{2x^2+x-6} \div \frac{3}{9-4x^2}$ [3]

(a)

$$\frac{3x}{2-3y} + \frac{6x}{9y^2-4}$$

$$= \frac{3x}{2-3y} + \frac{6x}{(3y)^2 - 2^2}$$

$$= \frac{3x}{2-3y} + \frac{6x}{(3y+2)(3y-2)}$$

$$= \frac{3x}{2-3y} - \frac{6x}{(3y+2)(2-3y)}$$

$$= \frac{3x(3y+2)}{(2-3y)(3y+2)} - \frac{6x}{(3y+2)(2-3y)}$$

$$= \frac{9xy + 6x - 6x}{(2-3y)(3y+2)}$$

$$= \frac{9xy}{(2-3y)(3y+2)}$$

(b)

$$\frac{x+2}{2x^2+x-6} \div \frac{3}{9-4x^2}$$

$$= \frac{x+2}{(2x-3)(x+2)} \div \frac{3}{3^2-(2x)^2}$$

$$= \frac{x+2}{(2x-3)(x+2)} \div \frac{3}{(3+2x)(3-2x)}$$

$$= \frac{x+2}{(2x-3)(x+2)} \times \frac{(3+2x)(3-2x)}{3}$$

$$= \frac{x+2}{(2x-3)(x+2)} \times \frac{-(3+2x)(2x-3)}{3}$$

$$= \frac{-(3+2x)}{3}$$